The Dedekind-Mertens Theorem for power series and more generally arbitrary algebras over R

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Abstract

In 1892 two mathematicians, R. Dedekind and F. Mertens, independently proved a generalization of Gauss' Lemma that works for any commutative ring R. Specifically they proved that if $f, g \in R[x]$, then there exists an integer k such that $c(f)^k c(g) = c(f)^{k-1} c(fg)$, where $c(f)$ denotes the content of the polynomial f. We generalize this formula to power series over a commutative Noetherian ring R, where the notion of content extends in the obvious fashion. We also show that the Dedekind-Mertens formula can hold (for power series) over certain non-Noetherian rings. In the second half of the talk we examine the notion of content that J. Ohm and D. Rush generalized to an arbitrary algebra over R (though it can behave quite badly). We examine certain properties an algebra may have with respect to this function – content algebra, weak content algebra, semicontent algebra (our own definition), and conditions when the Dedekind-Mertens formula holds for this function.

Keywords: Noetherian ring, Gauss' Lemma, Dedekind-Mertens formula.