The Zariski topology on sets of semistar operations

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Abstract

Let R be an integral domain with quotient field K. *Semistar operations* on R are a class of closure operations on the set of R-submodule of K. We endow the set $\mathrm{SStar}(R)$ of semistar operations on R with a natural topology (which we call the *Zariski topology*), in such a way that the set $\mathrm{Over}(R)$ of overrings of R becomes a subspace of $\mathrm{SStar}(R)$. We then use this topology to study three different subspaces: the set of semistar operations of finite type, of spectral semistar operations and of valutative semistar operations. We show that, if Δ is a compact set of finite-type properties, then its infimum (in the natural order) is still of finite type, and that the converse holds if the members of Δ are induced by localizations of R or by valuation domains. We also show that spectral and valutative operations have a very similar topological structure, while there are differences in their algebraic properties.! Moreover, we show that the sets of finite-type semistar operations, of finite-type spectral semistar operations and finite-type valutative semistar operations are spectral, that is, they are homeomorphic to the prime spectrum of some ring. – This is a joint work with Carmelo Finocchiaro and Marco Fontana.

Keywords: closure operation, localization, valuation domain.