The Zariski topology on sets of semistar operations

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Abstract

Let \( R \) be an integral domain with quotient field \( K \). Semistar operations on \( R \) are a class of closure operations on the set of \( R \)-submodule of \( K \). We endow the set \( \text{SStar}(R) \) of semistar operations on \( R \) with a natural topology (which we call the Zariski topology), in such a way that the set \( \text{Over}(R) \) of overrings of \( R \) becomes a subspace of \( \text{SStar}(R) \). We then use this topology to study three different subspaces: the set of semistar operations of finite type, of spectral semistar operations and of valutative semistar operations. We show that, if \( \Delta \) is a compact set of finite-type properties, then its infimum (in the natural order) is still of finite type, and that the converse holds if the members of \( \Delta \) are induced by localizations of \( R \) or by valuation domains. We also show that spectral and valutative operations have a very similar topological structure, while there are differences in their algebraic properties! Moreover, we show that the sets of finite-type semistar operations, of finite-type spectral semistar operations and finite-type valutative semistar operations are spectral, that is, they are homeomorphic to the prime spectrum of some ring. – This is a joint work with Carmelo Finocchiaro and Marco Fontana.

Keywords: closure operation, localization, valuation domain.