

The Survival Complex

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Abstract

We introduce a new way to associate a simplicial complex called the *survival complex* to a commutative semigroup with zero. Restricting our attention to the semigroup of monomials arising from an Artinian monomial ring, we determine that any such complex has an isolated point. Indeed, we show that there is exactly one isolated point essentially only in the case where the monomial ideal is generated purely by powers of the variables. This allows us to recover Beintema's result that an Artinian monomial ring is Gorenstein if and only if it is a complete intersection. A key ingredient of the translation between the pure power result and Beintema's result is given by the one-to-one correspondence we show between the so-called *truly isolated* points of our complex and the generators of the socle of the defining ideal. In another relation between the geometry of the complex and the algebra of the ring, we essentially give a correspondence between the nontrivial connected components of the complex and the factors of a fibre product representation of the ring. Finally, we explore algorithms for building survival complexes from specified isolated points. That is, we work to build the ring out of a description of the socle.

Keywords: simplicial complex, commutative semigroup, Artinian monomial ring, Gorenstein.