

# Some basic properties of primary decomposition

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## Abstract

Assume  $R$  is a commutative Noetherian ring. If  $R$  be a UFD (Unique Factorization Domain), then every (non-zero) element is a product of powers of distinct primes. When  $R = \mathbb{Z}$  for example, we have  $12 = 3 \cdot 4 = 3 \cdot (2^2)$ ; in terms of ideals, we have  $(12) = (3) \cap (2^2)$ . In general, the above factorizations are not available. However, every proper ideal  $I$  of  $R$  has a *primary decomposition*, in which  $I$  is expressed as an intersection of finitely many primary ideals (such as  $(12) = (3) \cap (2^2)$  over  $\mathbb{Z}$ ). More generally, one can study primary decompositions for modules. – In this talk, we go over some of the basic properties of primary decomposition. In particular, the compatibility, uniqueness, and linear growth properties will be discussed.

**Keywords:** Noetherian ring, factorization, primary decomposition.