

Schröder paths, partial horizontal strips, and symmetric functions

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Abstract

A partial horizontal strip in the skew shape μ/ν is a set of boxes, at most one in each column, such that the height of the boxes is weakly increasing to the right. The collection of numbers of boxes in each row in a partial horizontal strip form a partition, called the type of the partial horizontal strip.

A Schröder path is a lattice path from $(0, 0)$ to (n, n) using the steps north, northeast, and east that do not rise above the diagonal. The adjacent east steps of a Schröder path form a partition, call the type of the Schröder path.

We give a bijection between the set of all partial horizontal strips in the skew shape (n^n) $(n - 1, n - 2, \dots, 2, 1)$ and the set of all Schröder paths from $(0, 0)$ to (n, n) which preserves the type. We also give the formula for the number of Schröder paths of the given length and type.

Armstrong and Eu define a nonhomogeneous symmetric function, generalizing a construction of Pak and Postnikov. We define an elementary symmetric function analogue of Armstrong and Eu's symmetric function and show that the coefficients of this function when expanded in the complete elementary basis are given in terms of the type of partial horizontal strips.

Keywords: partial horizontal strip, Schröder path, symmetric function.