The incidence algebra of a poset and enumerations

Geir Agnarsson, George Mason University, Fairfax VA – 22030

Abstract

Let P be a locally finite poset; so every interval $[x, y] = \{z \in P : x \leq z \leq y\}$ is finite, and let $C(P)$ be the free $\mathbb{Z}$-module generated by all the intervals. In a fairly natural way one makes a coalgebra $C(P) = (C(P), \Delta, \epsilon)$, the dual of which $C^*(P) = I(P)$ turns out to be naturally a $\mathbb{Z}$-algebra, called the *incidence algebra of* P *over* $\mathbb{Z}$. We show how $I(P)$ generalizes the notion of a generating function for enumeration, and how one can obtain answers for various enumeration problems directly from $I(P)$. – This very natural generalization was first noted by Rota in 1964 and later more systematically by Goldman and Rota in 1970.

Keywords: poset, coalgebra, incidence algebra, generating function.