Some Combinatorial Types of Polytopes

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Abstract

The poset of faces of a polytope gives its "combinatorial type." Such a poset has several strong properties: It is an atomic lattice, its Möbius function is easily described (using Euler's relation), and so forth. However a complete characterization seems far out of reach. In the plane, any two convex n-gons are of the same combinatorial type. In three dimensions, where things are considerably more complicated, a complete characterization is nevertheless afforded by a theorem of Steinitz. In four and higher dimensions, no such characterization is known. Perhaps part of the difficulty arises because of inability to view higher-dimensional polytopes in the way that we can visualize three-dimensional ones. This talk will present some combinatorial types of high-dimensional polytopes in forms for which the combinatorial structures can be easily (more or less) understood.

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