

Universal lying-over rings

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Abstract

A (commutative unital) ring R is said to satisfy universal lying over (ULO) if each injective ring homomorphism $R \rightarrow T$ satisfies the lying-over property, that is the induced map on prime ideals $\text{Spec}(T) \rightarrow \text{Spec}(R)$ given by $P \mapsto P \cap R$, is onto. We present both sufficient conditions and necessary conditions for R to satisfy ULO. Using the sufficient conditions we construct rings of arbitrary Krull dimension satisfying ULO. We also show that the necessary conditions are not sufficient by constructing rings of arbitrary Krull dimension that satisfy these conditions but do not satisfy ULO.

Keywords: Commutative ring, lying over property (LOP), Krull dimension.