A quasisymmetric function for matroids

Louis J. Billera, Department of Mathematics, Cornell University, Ithaca, NY 14853.

Abstract

We consider the formal power series $F(M)$ defined as the weight enumerator of all generic positive integral forms over the bases of a matroid $M$. This association gives a Hopf algebra map from the Hopf algebra of (isomorphism classes) of matroids to QSym, the Hopf algebra of all quasisymmetric functions, displaying interesting matroid invariants when expressed in terms of the fundamental basis for QSym.

More interesting is the fact that $F(M)$ can be used to detect when the matroid polytope $P(M)$ can be decomposed into the matroid polytopes of degenerations of $M$. The proof of this depends on a result of Lawrence. Such decompositions have been of interest to Lafforgue in his study of compactifications of certain quotients of Grassmanians. He showed that the lack of such a decomposition implies the matroid has only a finite number of vector representations up to projective equivalence.

This is joint work with Victor Reiner and Ning Jia.

Keywords: matroid, Hopf algebra, polytope, Grassmanian manifold.