Moreau-type characterizations of polar cones

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Abstract

A theorem of Moreau (1962) states that given a closed convex cone $C$ and its (negative) polar cone $C^\circ$ in a real Hilbert space $H$, vectors $y \in C$ and $z \in C^\circ$ are metric projections of a vector $u \in H$ on $C$ and $C^\circ$, respectively, if and only if they satisfy the following conditions: $y$ and $z$ are orthogonal and $u = y + z$. We show that these conditions provide characteristic properties of polar cones $C$ and $C^\circ$ in the family of pairs of convex subsets of $H$ or $\mathbb{R}^n$. A related result on separation of $C$ a face of $C^\circ$ in $\mathbb{R}^n$ is proved.

Keywords: convex cone, polar cone, Hilbert space.