

Moreau-type characterizations of polar cones

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Abstract

A theorem of Moreau (1962) states that given a closed convex cone C and its (negative) polar cone C° in a real Hilbert space H , vectors $y \in C$ and $z \in C^\circ$ are metric projections of a vector $u \in H$ on C and C° , respectively, if and only if they satisfy the following conditions: y and z are orthogonal and $u = y + z$. We show that these conditions provide characteristic properties of polar cones C and C° in the family of pairs of convex subsets of H or \mathbb{R}^n . A related result on separation of C a face of C° in \mathbb{R}^n is proved.

Keywords: convex cone, polar cone, Hilbert space.