Algebraic and combinatorial interpretations of tight closure theory on Stanley-Reisner rings

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Abstract

Let R be a Noetherian commutative ring, $x \in R$, and I be an ideal of R. We say x is in the tight closure of I (denoted I^*) if $\exists\ c \in R^\circ$ such that for all $q \gg 0$, $cx^q \in I^{[q]}$. ($I^{[q]}$ denotes the q-th Frobenius power of the ideal I, and $q = p^e$ for any $e \in \mathbb{N}_0$.) Given an ideal $J \subseteq I$, we say that J is a $*$-reduction of I if $J^* = I^*$ and the reduction is minimal if for ideals $K \subsetneq J$, $I \not\subset K^*$. The $*$-spread of I is the minimal number of generators of a minimal $*$-reduction I, and $*$-core(I) is the intersection of all minimal $*$-reductions of I. – For any Stanley-Reisner ring $k[\Delta]$ over the variable set $\{x_1, \dots, x_n\}$, we define $\mathfrak{m}$ to be the maximal ideal of $k[\Delta]$ generated by the images of the variables. We discuss the above tight closure notions on Stanley-Reisner rings and show that the $*$-spread of $\mathfrak{m}$ is $\dim k[\Delta]$ and provide bounds $*$-core($\mathfrak{m}$).

Keywords: Noetherian ring, tight closure, Frobenius power, $*$-spread of an ideal, Stanley-Reisner ring.