A New Perspective on the $\mathcal{G}$-Invariant of a Matroid

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Abstract

The $\mathcal{G}$-invariant of a matroid was introduced by Derksen (2009), who showed that it properly generalizes the Tutte polynomial. The Tutte polynomial is universal among invariants that satisfy deletion-contraction rules; analogously, Derksen and Fink (2010) showed that the $\mathcal{G}$-invariant is universal among invariants that satisfy an I/E-like property on matroid base polytopes. – In this joint work with Joseph Kung, we give a new view of this invariant and explore its implications. We show that the $\mathcal{G}$-invariant of a matroid M of rank r is equivalent to having, for each sequence $a_0, a_1, \ldots, a_r$, the number of flags of flats $X_0 \subset X_1 \subset \cdots \subset X_r$ of M with $|X_0| = a_0$ and $|X_i - X_{i-1}| = a_i$ for $i > 0$. With this view, we can determine the effect of some matroid constructions, such as taking q-cones, on the $\mathcal{G}$-invariant, and we can identify some of the information that the $\mathcal{G}$-invariant picks up that the Tutte polynomial does not, such as the number of circuits and cocircuits of any given size, and the number of cyclic flats of any given rank and size. From its $\mathcal{G}$-invariant, we can tell whether a matroid is a free product of two other matroids (other than free extensions and coextensions). Also, the $\mathcal{G}$-invariant of a matroid can be reconstructed from the multi-set of $\mathcal{G}$-invariants of the restrictions to hyperplanes. Still, strengthening what J. Eberhardt proved for the Tutte polynomial, the $\mathcal{G}$-invariant is determined by relatively little information, namely, by just the isomorphism type of the lattice of cyclic flats along with the rank and size of each cyclic flat.

Keywords: matroid, Tutte polynomial, matroid base polytope.