Parking functions & friends

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Abstract

Imagine a one-way street with n parking spots and a cliff at its end. We'll give the first parking spot the number 1, the next one number 2, etc., down to the last one, number n. Initially they're all free, but there are n cars approaching the street, and they'd all like to park. To make life interesting, every car has a parking preference, and we record the preferences in a sequence; e.g., if $n = 3$, the sequence $(2, 1, 1)$ means that the first car would like to park at spot number 2, the second car prefers parking spot number 1, and the last car would also like to part at number 1. The street is very narrow, so there is no way to back up. Now each car enters the street and approaches its preferred parking spot; if it is free, it parks there, and if not, it moves down the street to the first available spot. We call a sequence a *parking function* if all cars end up finding a parking spot, i.e., none fall off the cliff. E.g., the sequence $(2, 1, 1)$ is a parking function, whereas the sequence $(1, 3, 3, 4)$ is not. – A beautiful theorem due to Konheim and Weiss says that there are precisely $(n + 1)^{n-1}$ parking functions of length n. We will hint at a proof of this theorem and illustrate how it allows us to connect parking functions to seemingly unrelated objects, which happen to exhibit the same counting pattern: a certain set of hyperplanes in n-dimensional space first studied by Shi, and a certain family of mixed graphs, which we introduced in recent joint work with Ana Berrizbeitia, Michael Dairyko, Claudia Rodriguez, Amanda Ruiz, and Schuyler Veeneman.

Keywords: parking function, counting patterns, hyperplanes.