

Complete Intersection Affine Semigroup Rings Arising from Posets

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Abstract

This is a talk about a class of directed graphs defined by posets that satisfy a certain algebraically defined property. It is not a ring theory talk. We apply theorems of Fischer, Morris and Shapiro on affine semigroup rings to show that if a certain affine semigroup ring defined by a poset is a complete intersection, then the poset is either unicyclic or contains a chain, the removal of which increases the number of connected components of the Hasse diagram. This is the converse of a theorem of Boussicault, Feray, Lascoux and Reiner. We show that the rows of a matrix of relations for the affine semigroup form a weakly fundamental cycle basis consisting of circuits of the digraph given by the Hasse diagram of the poset, but give an example of such a matrix that is not totally unimodular. We also show that the number of edges of the Hasse diagram of a poset for which the affine semigroup ring is a complete intersection is bounded above by twice the number of vertices minus 4.

Keywords: directed graph, semigroup, affine semigroup ring, poset, Hasse diagram.