The geometry of the quotient stack arising from a stacky fan

David A. Johannsen, George Mason University, Fairfax VA – 22030

Abstract

A quotient stack $[Z/G]$ is a geometric object that models the quotient of a space Z by the action of a Lie group G while carrying additional structure at the singularities. Quotient stacks generalize toric varieties, and thus constitute a broad and important class of geometric spaces. In this dissertation, we will exploit a construction given by Borisov, Chen, and Smith that allows one to construct a quotient stack from a particular combinatorial object, called a stack fan. Our program is to deduce geometric features of the quotient stack from the stacky fan.

Our main results are to determine the component group of the Lie group G from the combinatorics of the stacky fan. In particular, we will give a necessary and sufficient condition on the stacky fan for the corresponding group G to be connected. We will also give a characterization of all the inertia groups of the quotient stack, in terms of the combinatorics of the stacky fan.

Finally, we will turn our attention to the stacky fans that give rise to weighted projective spaces (and fake weighted projective spaces), a very important class of toric varieties. In particular, we will give a characterization of stacky fans that correspond to weighted projective space. In the case of 2-dimensions, we will be able to give an explicit and complete description of

the resulting quotient stack, in terms of the greatest common divisor of positive integers associated to the stacky fan.

Keywords: symplectic geometry, quotient stack, toric variety.