Induced subgraphs of $\mathbb{Z}^2$ and related extremal graphs

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Abstract

We prove in a new way that the maximum number of edges an induced subgraph of the 2-grid $\mathbb{Z}^2$ on $n$ vertices is given by $E_2(n) = \lfloor 2n - \sqrt{2n} \rfloor$. This was first proved by Harary and Harborth in 1976 by induction. Our method suggests some possible ways to obtain analog results in higher dimensions via projections and the Box Theorem by Bollobás and Thomas from 1995. – We discuss some of these key tools and related results in discrete geometry.

Keywords: grid graph, induced subgraph, Box Theorem.