

The free splice of matroids

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Abstract

Given matroids $M(A)$ and $N(B)$ such that the restrictions $M|(A \cap B)$ and $N|(A \cap B)$ are equal, an *amalgam* of M and N is a matroid L on $A \cup B$ such that $L|A = M$ and $L|B = N$. Amalgams might not exist, and even when they do, there might be no freest amalgam. In this talk we consider the “twisted” version of this situation, in which the restriction $N|(A \cap B)$ is equal to the contraction $M/(A - B)$. In this case we define a *splice* of M and N to be a matroid L on $A \cup B$ such that $L|A = M$ and $L/(A - B) = N$. In contrast to the situation for amalgams, it turns out splices always exist and, furthermore, there is a freest splice, which we call the *free splice* of M and N . (In the case of disjoint A and B , the free splice is the free product, introduced by Henry Crapo and William Schmitt in 2005.) We show how to construct the free splice, as a certain Higg’s lift, and give cryptomorphic descriptions of it, in terms of bases, independent set, rank function, flats and cyclic flats. We show that minors of free splices are free splices of higgs lifts of corresponding minors, and we characterize, in terms of cyclic flats, matroids that are irreducible with respect to free splice. We describe the associativity properties of free splice and examine its interaction with some other matroid operations. – This is joint work with Joe Bonin.

Keywords: matroid, amalgam, splice.