

Colloquial Usage Again, we point out that colloquial usage is frequently not precise. For example, the statement “All students in finite mathematics courses do not fail” is strictly interpreted to mean that $\forall x \in U$ [x does not fail] with $U =$ the set of students in finite mathematics courses. This means, of course, that no one fails. On the other hand, some might loosely interpret this to mean that not every student in finite mathematics courses fails. This is the statement $\exists x \in U$ [x does not fail]. We require precision of language in mathematics and do *not* accept the second interpretation as correct.

Practice Problems 12.6

- Write the following statement symbolically and write out its negation in English: “There exists a flower that can grow in sand and is not subject to mold.”
- For the universe $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, determine the truth values of the following statements.

(a) $\forall x (x > 2)$	(b) $\exists x (x > 2)$
(c) $\forall x (x^2 < 100)$	(d) $\exists x [(x - 1 = 4) \wedge (3x + 5 = 20)]$
- Consider the universe for both variables x and y to be the set of nonnegative integers $\{0, 1, 2, 3, 4, \dots\}$. Write out the statements in English and determine the truth value of each.

(a) $\forall x \forall y [x < y]$	(b) $\forall x \exists y [x < y]$
(c) $\forall y \exists x [x < y]$	(d) $\exists x \forall y [x < y]$
(e) $\exists x \exists y [x < y]$	(f) $\forall x \forall y [(x < y) \vee (y < x)]$

Exercises 12.6

- Let $U = \{1, 2, 3, 4, 5, 6\}$. Determine the truth value of

$$p(x) = [(x \text{ is even}) \text{ or } (x \text{ is divisible by } 3)]$$
 for the given values of x .

(a) $x = 1$	(b) $x = 4$
(c) $x = 3$	(d) $x = 6$
(e) $x = 5$	
- Determine the truth value of $p(x)$ for the values of x chosen from the universe of all letters of the alphabet where

$$p(x) = [(x \text{ is a vowel}) \text{ and } (x \text{ is in the word ABLE})].$$

(a) $x = a$	(b) $x = d$
(c) $x = b$	(d) $x = i$
- Consider the universe of all college students. Let $p(x)$ denote the open statement “ x takes a writing course.”
 - Write the statement “Every college student takes a writing course” in symbols.
 - Write the statement “Not all college students take a writing course” symbolically.
 - Write the statement “Every college student does not take a writing course” symbolically.
 - Do any of the statements in (a), (b), or (c) imply one another? Explain.
- Recently, as the Amtrak train pulled into the Baltimore station, the conductor announced, “All doors do not open.” Since passengers were permitted to get off at the station, what do you think the conductor meant to say?
- An alert California teacher chided “Dear Abby” (*Baltimore Sun*, March 1, 1989) for her statement “Confidential to Eunice: All men do not cheat on their wives.” Let $p(x)$ be the statement “ x cheats on his wife” in the universe of all men. Write out Abby’s statement symbolically. Rewrite the statement using the existential quantifier. Do you think the statement is TRUE or FALSE? What do you think Abby really meant to say?
- Consider the universe of all orange juice. Write the following symbolically, using $p(x) =$ “ x comes from Florida.”
 - All orange juice does not come from Florida.
 - Some orange juice comes from Florida.
 - Not all orange juice comes from Florida.
 - Some orange juice does not come from Florida.
 - No orange juice comes from Florida.
 - Are any of the statements (a) through (e) equivalent to one another? Explain.

7. Let the universe be all university professors. Let $p(x)$ be the open statement "x likes poetry." Write the following statements symbolically.
- (a) All university professors like poetry.
 - (b) Some university professors do not like poetry.
 - (c) Some university professors like poetry.
 - (d) Not all university professors like poetry.
 - (e) All university professors do not like poetry.
 - (f) No university professors like poetry.
 - (g) Are any of the statements in (a) through (f) equivalent? Explain.

8. Let $U = \{0, 1, 2, 3, 4\}$. Let

$$p(x) = [x^2 > 9].$$

Find the truth value of

- (a) $\exists x p(x)$
- (b) $\forall x p(x)$

9. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. Let

$$p(x) = [(x \text{ is prime}) \rightarrow (x^2 + 1 \text{ is even})].$$

Find the truth value of

- (a) $\exists x p(x)$
- (b) $\forall x p(x)$

10. Let $U = \{3, 4, 5, 6, \dots\}$. Let

$$p(x) = [(x \text{ is prime}) \rightarrow (x^2 + 1 \text{ is even})].$$

Find the truth value of

- (a) $\exists x p(x)$
- (b) $\forall x p(x)$

11. Let the universe consist of all nonnegative integers. Let $p(x)$ be the statement "x is even." Let $q(x)$ be the statement "x is odd." Determine the truth value of

- (a) $\forall x [p(x) \vee q(x)]$
- (b) $[\forall x p(x)] \vee [\forall x q(x)]$
- (c) $\exists x [p(x) \vee q(x)]$
- (d) $\exists x [p(x) \wedge q(x)]$
- (e) $\forall x [p(x) \wedge q(x)]$
- (f) $[\exists x p(x)] \wedge [\exists x q(x)]$
- (g) $\forall x [p(x) \rightarrow q(x)]$
- (h) $[\forall x p(x)] \rightarrow [\forall x q(x)]$

12. Let the universe consist of all real numbers. Let

$$p(x) = (x \text{ is positive})$$

and

$$q(x) = (x \text{ is a perfect square}).$$

Determine the truth value of

- (a) $\forall x p(x)$
- (b) $\forall x q(x)$
- (c) $\exists x p(x)$
- (d) $\exists x q(x)$
- (e) $\exists x [p(x) \rightarrow q(x)]$
- (f) $\exists x [q(x) \rightarrow p(x)]$
- (g) $\forall x [p(x) \rightarrow q(x)]$
- (h) $\forall x [q(x) \rightarrow p(x)]$
- (i) $\forall x p(x) \rightarrow \exists x q(x)$
- (j) $[\forall x p(x)] \rightarrow [\forall x q(x)]$

13. Negate the following statements.

- (a) Every dog has its day.
- (b) Some men fight wars.

- (c) All mothers are married.
- (d) For every pot, there is a cover.
- (e) Not all children have pets.
- (f) Not every month has 30 days.

14. Negate each statement by changing existential quantifiers to universal quantifiers, or vice versa.

- (a) Every stitch saves time.
- (b) All books have hard covers.
- (c) Some children are afraid of snakes.
- (d) Not all computers have a hard disk.
- (e) Some chairs do not have arms.

15. Consider the universe of nonnegative integers = $\{0, 1, 2, 3, 4, \dots\}$. Write the English sentence for each symbolic statement. Determine the truth value of each statement. If the statement is FALSE, give a counterexample and write its negation out in words.

- (a) $\forall x \forall y [x + y > 12]$
- (b) $\forall x \exists y [x + y > 12]$
- (c) $\exists x \forall y [x + y > 12]$
- (d) $\exists x \exists y [x + y > 12]$

16. Consider the universe of all subsets of the set $A = \{a, b, c\}$. Let the variables x and y denote subsets of A . Find the truth value of each of the following statements and explain your answer. If the statement is FALSE, give a counterexample.

- (a) $\forall x \forall y [x \subseteq y]$
- (b) $\forall x \forall y [(x \subseteq y) \vee (y \subseteq x)]$
- (c) $\exists x \forall y [x \subseteq y]$
- (d) $\forall x \exists y [x \subseteq y]$

17. Let the universe for both variables x and y be the set $\{1, 2, 3, 4, 5, 6\}$. Let $p(x, y) = "x \text{ divides } y."$ Give the truth values of each of the following statements; explain your answer, and give a counterexample in case the statement is FALSE.

- (a) $\forall x \forall y p(x, y)$
- (b) $\forall x \exists y p(x, y)$
- (c) $\exists x \forall y p(x, y)$
- (d) $\exists y \forall x p(x, y)$
- (e) $\forall y \exists x p(x, y)$
- (f) $\forall x p(x, x)$

18. If $p(x)$ denotes " $x \in S$ " and $q(x)$ denotes " $x \in T$," describe the following using logical statement forms with $p(x)$ and $q(x)$.

- (a) $x \in S' \cup T$
- (b) $x \in S \oplus T'$
- (c) $x \in S' \cap T'$
- (d) $x \in (S \cup T)'$

19. Let

$$U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13\},$$

let $S = \{x \in U : x \geq 8\}$, and let $T = \{x \in U : x \leq 10\}$.

- (a) What implication must be TRUE if and only if S is a subset of T ?
- (b) Is S a subset of T ? Explain.

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