Replacement vs. No Replacement

1. Five numbers are chosen at random from the set $\{1, 2, 3 \ldots 13\}$ **with replacement**.
   Find the probability that all are even.

Solution: The outcomes from this experiment are strings of length 5, where repeats are allowed. There are 13^5 different strings so we have $n(S) = 13^5$.
The number of strings with all even digits is 6^5, so we have $n(E) = 6^5$

$$P(all\ even) = \frac{6^5}{13^5} = .02094$$

Similarly, we find the probability that all 5 digits are odd:

$$P(all\ odd) = \frac{7^5}{13^5} = .04527$$

2. Five numbers are chosen at random from the set $\{1, 2, 3 \ldots 13\}$ **without replacement**.
   Find the probability that all are even.

Solution: The outcomes from this experiment are collections of 5 distinct numbers. The 5 numbers can be chosen all at once or one at a time. (*see below for alternate solution)

If we choose them **all at once**, $n(S) = \binom{13}{5}$. The number of outcomes with all even digits is $n(E) = \binom{6}{5}$. From this we get, $P(all\ even) = \frac{\binom{6}{5}}{\binom{13}{5}} = .00466$

Similarly, $P(all\,odd) = \frac{\binom{7}{5}}{\binom{13}{5}} = .01632$

Conclusion 1: Comparing the probabilities we see that the experiments **with replacement** and **without replacement** are different.

* Choose 5 numbers **one at a time** at random from the set $\{1, 2, 3 \ldots 13\}$ **without replacement.**

Here, the outcomes are strings of length 5 with no repeated digits.

$n(S) = 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9$ and $n(E) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2$

$$P(all\ even) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9} = .00466 \quad \text{and} \quad P(all\ odd) = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9} = .01632$$

These probabilities are exactly the same as the ones we got when choosing the numbers **all at once**.

Conclusion 2: Whether we choose the numbers all at once (no order) or one at a time (in order), we arrive at the same probability. We can consider those two experiments as equivalent.