

### Problem 14 (Page 435) Extended

See Matlab diary for calculations.

1. Suppose in a given year we have 1% of the population chronically insane, 12% mildly insane and 87% sane. What percent of the population is expected to be chronically insane after one year? After two years?

Answer: Let  $X_0 = \begin{bmatrix} .01 \\ 0 \\ .87 \\ .12 \end{bmatrix}$ . Calculate  $T * X_0 = \begin{bmatrix} .0124 \\ .0209 \\ .8526 \\ .1141 \end{bmatrix}$  and  $T^2 * X_0 = \begin{bmatrix} .0147 \\ .0414 \\ .8355 \\ .0184 \end{bmatrix}$

1.24% after one year; 1.47% after 2 years;

2. What percent of mildly insane people will be chronically insane after one year? After two years?

Answer: Use  $T$  and  $T^2$  to get the information about people who start in state IV. The percent is taken from column 4, row 1 of  $T$ : 2% after one year; From the same position in  $T^2$ : 3.9% after two years

3. Suppose 200 people are mildly insane. How many of these people will be chronically insane after two years?

Answer: (use information problem 2 above)  $3.9\% \text{ of } 200 = 7.8$  people

4. Find the percent of mildly insane people who will eventually become chronically insane.

Answer: Calculate the stable matrix for this process.

40% of mildly insane people eventually end up chronically insane.

```
>> T=[1 0 0 .02;0 1 .01994 .03;0 0 .98 0;
0 0 .00006 .95]
T =
```

```
1.0000    0    0    0.0200
    0    1.0000    0.0199    0.0300
    0    0    0.9800    0
    0    0    0.0001    0.9500
```

```
>> X0=[.01;0; .87;.12]
X0 =
```

```
0.0100
    0
0.8700
0.1200
```

```
>> T*X0
ans =
```

```
0.0124
0.0209
0.8526
0.1141
```

```
>> T^2*X0
ans =
```

```
0.0147
0.0414
0.8355
0.1084
```

```
>> T^2
ans =
```

```
1.0000    0    0.0000    0.0390
    0    1.0000    0.0395    0.0585
    0    0    0.9604    0
    0    0    0.0001    0.9025
```

```
>>% To find the stable matrix
>>% partition T and identify the
>>% submatrices S and R
```

```
>> S=T(1:2,3:4)
```

```
S =
```

```
    0    0.0200
0.0199    0.0300
```

```
>> R=T(3:4,3:4)
```

```
R =
```

```
0.9800    0
0.0001    0.9500
```

```
>> S*inv(eye(2)-R)
```

```
ans =
```

```
0.0012    0.4000
0.9988    0.6000
```

```
>> stableT=[1 0 .0012 .4;0 1 .9988 .6;0
0 0 0 0 0]
```

```
stableT =
```

```
1.0000    0    0.0012    0.4000
    0    1.0000    0.9988    0.6000
    0    0    0    0
    0    0    0    0
```