

**Math 671–001 (Fourier Analysis)**  
**Fall 2012**

**Instructor:** David Walnut, Planetary Hall (formerly Science and Technology I), room 261

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**Office hours:** MW 3:30–5:00 and by appointment.

**Text:** Elias M. Stein and Rami Shakarchi, *Fourier Analysis: An Introduction*, Princeton Lectures in Analysis I, Princeton University Press (2003).

**Prerequisites:** Advanced calculus (Math 315 or equivalent).

**Topics:** The course will cover some subset of the following topics. The corresponding sections in Stein and Shakarchi are given.

1. Historical Motivation
  - (a) Wave Equation (Ch. 1, Sec. 1)
  - (b) Heat Equation (Ch. 1, Sec. 2)
2. Fourier Series
  - (a) Definition and Uniqueness (Ch. 2, Sec. 1, 2)
  - (b) Approximate Identities and Convergence (Ch. 2, Sec. 3-5)
  - (c) Hilbert Space methods (orthogonality,  $L^2$  Convergence, Parseval/Plancherel formula) (Ch. 3, Sec. 1)
  - (d) Applications and topics (Ch. 4)
3. The Fourier transform on  $\mathbf{R}$  and  $\mathbf{R}^d$ 
  - (a) The Schwartz Class  $\mathcal{S}(\mathbf{R}^d)$ . (Ch. 5, Sec. 1.1-1.3)
  - (b) Definition and properties of the Fourier transform on  $\mathcal{S}$ . (Ch. 5, Sec. 1.4; Ch. 6, Sec. 1-2)
  - (c) Fourier Inversion (Ch. 5, Sec. 1.5; Ch. 6, Sec. 2)
  - (d) Extension to other function classes. (Ch. 5, Sec. 1.6-1.8; Ch. 6, Sec. 2)
  - (e) Applications and topics (Ch. 5, Sec. 2-3; Ch. 6, Sec. 3)
4. Sampling theory and related topics
  - (a) Bandlimited functions
  - (b) Classical Sampling
  - (c) Introduction to frames and Riesz bases of exponentials
  - (d) Irregular sampling and the 1/4-theorem
  - (e) Sampling in higher dimensions
  - (f) Applications and topics
5. The Radon transform (Ch. 6, Sec. 5)
  - (a) Definition and basic properties
  - (b) Inversion formula for the RT
  - (c) Applications and topics

**Grading:**

*Homework:* There will be regular homework assignments given throughout the semester. All assignments are to be typed up using some kind of word-processing software. MS Word is adequate to handle any symbols that you must type, but you can use any software you like. Any flavor of TeX is ideal for this task. Homework counts for 3/4 of your final grade.

*Exams:* The remaining 1/4 of your grade will be based on a take-home midterm (1/8) and an in-class final exam (1/8). Precise dates and coverage for these exams will be announced.