Exercise 4 – Traveling waves from standing waves. (a) Show that if F and G are twice differentiable functions then the function

$$u(x,t) = F(x+t) + G(x-t) \tag{1}$$

is a solution to the wave equation $u_{tt} = u_{xx}$. This solution is known as the *travelling wave solution* to the wave equation.

(b) Show that the standing wave solution derived in class, namely

$$u(x,t) = \sum_{m=1}^{\infty} (a_m \cos(mt) + b_m \sin(mt)) \sin(mx)$$

$$f(x) = u(x,0) = \sum_{m=1}^{\infty} a_m \sin(mx) \qquad g(x) = u_t(x,0) = \sum_{m=1}^{\infty} mb_m \sin(mx)$$

satisfies (1) with

$$F(x) + G(x) = f(x) \qquad \text{and} \qquad F'(x) - G'(x) = g(x).$$

(Hint: For these calculations you may manipulate all infinite series formally without worrying about convergence.)