9.9. Curl of a Vector Field.

A. Curl.

1. Given a vector field \( \mathbf{v}(x, y, z) = [v_1(x, y, z), v_2(x, y, z), v_3(x, y, z)] \) we define the curl of \( \mathbf{v} \), denoted \( \text{curl} \mathbf{v} \), symbolically by

\[
\text{curl} \mathbf{v} = \nabla \times \mathbf{v} = \begin{vmatrix}
i & j & k \\
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
v_1 & v_2 & v_3
\end{vmatrix}.
\]

2. If \( \mathbf{v} \) is the velocity field of a fluid, then \( \text{curl} \mathbf{v}(P) \) is a vector measuring the tendency of the fluid to rotate about the point \( P \). \( \text{curl} \mathbf{v} \) is independent of the coordinate system.

3. Given any scalar field \( f \), and vector field \( \mathbf{v} \), \( \text{curl}(\nabla f) = 0 \) and \( \text{div}(\text{curl} \mathbf{v}) = 0 \).

B. Curl for Rigid Body Motion.

If a rigid body is rotating about an axis with angular speed \( \omega > 0 \), then the velocity field \( \mathbf{v} \) if the body satisfies \( \text{curl} \mathbf{v} = 2\mathbf{w} \) where \( \mathbf{w} \) is the vector with magnitude \( \omega \) and direction along the axis of rotation according to the right-hand rule.