Exercise 9.1.16.

Solution:

\[
(3a - 5b) + 2c = (3[2, -1, 0] - 5[-4, 2, 5]) + 2[0, 0, 3] \\
= ([6, -3, 0] - [-20, 10, 25]) + [0, 0, 6] \\
= [26, -13, -25] + [0, 0, 6] \\
= [26, -13, -19]
\]

\[
3a + (-5b + 2c) = 3[2, -1, 0] + (-5[-4, 2, 5] + 2[0, 0, 3]) \\
= [6, -3, 0] + ([20, -10, -25] + [0, 0, 6]) \\
= [6, -3, 0] + [20, -10, -19] \\
= [26, -13, -19]
\]

Exercise 9.1.30.

Solution:

The resultant vector is

\[
[3, 1, 7] + [4, 4, 5] + [3, 2, c] = [10, 7, 12 + c].
\]

Since parallel to the xy-plane means that the z component of the vector is zero, this means that \( c = -12 \).

Exercise 9.1.34.

Solution:

The unit vector in the “southwest” direction is \((-1/\sqrt{2})i - (1/\sqrt{2})j\). Hence \( \mathbf{v}_A = (-500/\sqrt{2})i - (500/\sqrt{2})j \). The unit vector in the “northwest” direction is \((-1/\sqrt{2})i + (1/\sqrt{2})j\). Hence \( \mathbf{v}_B = (-400/\sqrt{2})i + (400/\sqrt{2})j \). Therefore,

\[
\mathbf{v} = \mathbf{v}_B - \mathbf{v}_A = (100/\sqrt{2})i + (900/\sqrt{2})j
\]

which translates to a speed of \( |\mathbf{v}| \approx 640 \text{ mph} \) in a direction of \( \theta = \tan^{-1}(9) \approx 83.7^\circ \) north of east or about 6.3° east of north.
Exercise 9.2.16.

Solution:

\[ \mathbf{a} \cdot \mathbf{b} = [2, 1, 4] \cdot [-4, 0, 3] = 4, \quad |\mathbf{a}| = (4+1+16)^{1/2} = \sqrt{21}, \quad \text{and} \quad |\mathbf{b}| = (16+0+9)^{1/2} = 5. \]

Since \( \sqrt{21} \cdot 5 \approx 22.9 \), \( |\mathbf{a} \cdot \mathbf{b}| \leq |\mathbf{a}||\mathbf{b}| \) is verified.

\[ |\mathbf{a} + \mathbf{b}| = \sqrt{21} + 5 \approx 9.6. \]

Hence \( |\mathbf{a} + \mathbf{b}| \leq |\mathbf{a}| + |\mathbf{b}| \) is verified.

\[ |\mathbf{a} - \mathbf{b}| = \sqrt{38}. \]

This is the distance from the plane to the origin.

Exercise 9.2.32.

Solution:

Consider a triangle with sides \( a, b, \) and \( c \) with \( \gamma \) the angle opposite side \( c \). The Law of Cosines says that \( a^2 + b^2 + 2ab \cos \gamma = c^2 \). If we think of the sides of the triangle as vectors, then the sides are \( |\mathbf{a}|, |\mathbf{b}|, \) and \( |\mathbf{a} - \mathbf{b}| \) and \( \gamma \) is the angle between \( \mathbf{a} \) and \( \mathbf{b} \). Hence

\[ |\mathbf{a} - \mathbf{b}|^2 = (\mathbf{a} - \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \mathbf{a} \cdot \mathbf{a} - 2\mathbf{a} \cdot \mathbf{b} + \mathbf{b} \cdot \mathbf{b} = |\mathbf{a}|^2 + |\mathbf{b}|^2 - 2|\mathbf{a}||\mathbf{b}| \cos \gamma \]

which is precisely the Law of Cosines.

Exercise 9.2.36.

Solution:

A normal vector to the plane given by \( 5x + 2y + z = 10 \) is \( \mathbf{n} = [5, 2, 1] \). If the point \( P = (a, b, c) \) is on the plane then \( 5a + 2b + c = 10 \). Letting \( \mathbf{r} = \overrightarrow{OP} = [a, b, c] \), then the projection of \( \mathbf{r} \) onto \( \mathbf{n} \) is

\[ \frac{\mathbf{n} \cdot \mathbf{r}}{||\mathbf{n}||} = \frac{[5, 2, 1] \cdot [a, b, c]}{(25 + 4 + 2)^{1/2}} = \frac{5a + 2b + c}{\sqrt{30}} = \frac{10}{\sqrt{30}} \approx 1.83. \]

This is the distance from the plane to the origin.

Exercise 9.3.30.

Solution:

Because of the direction of rotation, the vector \( \mathbf{w} \) defining the motion has direction \( \left( \frac{1}{\sqrt{2}} \right) \mathbf{i} + \left( \frac{1}{\sqrt{2}} \right) \mathbf{j} \) (that is, it points into the first quadrant of the \( xy \)-plane while its \( \mathbf{k} \) component is zero). The magnitude of \( \mathbf{w} \) is the angular speed so that
\( w = (5/\sqrt{2})i + (5/\sqrt{2})j \). Letting \( r \) be the position vector of the point in question gives \( r = 4i + 2j - 2k \). Therefore,

\[
\mathbf{w} \times \mathbf{r} = -\frac{5}{\sqrt{2}}(\mathbf{i} + \mathbf{j}) \times 2(\mathbf{2i} + \mathbf{j} - \mathbf{k}) = \frac{10}{\sqrt{2}}(\mathbf{i} - \mathbf{j} + \mathbf{k}).
\]

Therefore the speed of the particle is \( |\mathbf{w} \times \mathbf{r}| = (10\sqrt{3})/\sqrt{2} \approx 12.25 \text{ sec}^{-1} \) and the direction is along the vector \( \mathbf{i} - \mathbf{j} + \mathbf{k} \).

Exercise 9.3.36.
Solution:

Letting \( P = (2, 1, 3) \), \( Q = (4, 4, 5) \), and \( R = (1, 6, 0) \), we have that a normal vector to the plane is

\[
\mathbf{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = [2, 3, 2] \times [-1, 5, -3] = -19\mathbf{i} + 4\mathbf{j} + 13\mathbf{k}.
\]

The equation of the plane then is \(-19x + 4y + 13z = c\). Plugging the point \( R \) into this equation gives \(-19(1) + 4(6) + 13(0) = 5 = c\). Hence the equation of the plane is \(-19x + 4y + 13z = 5\).

Exercise 9.3.38.
Solution:

Letting \( P = (0, 2, 1) \), \( Q = (4, 3, 0) \), \( R = (6, 6, 5) \), and \( S = (4, 7, 8) \), the volume of the parallelepiped whose sides are given by the vectors \( \overrightarrow{PQ}, \overrightarrow{PR}, \) and \( \overrightarrow{PS} \) is the absolute value of the scalar triple product

\[
(\overrightarrow{PQ} \; \overrightarrow{PR} \; \overrightarrow{PS}) = \begin{vmatrix} 4 & 1 & -1 \\ 6 & 4 & 4 \\ 4 & 5 & 7 \end{vmatrix} = -8.
\]

Since the volume of a tetrahedron is 1/6 the volume of the parallelepiped with the same edges, the volume of the tetrahedron is 4/3.