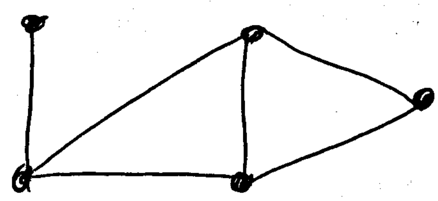
MATH 290 - EXAM 6 - SOLUTIONS

1.



There are many other correct answers

2. (a) $f = \{(a,1), (a,2), (b,3), (c,4)\}$

There are many other correct answers

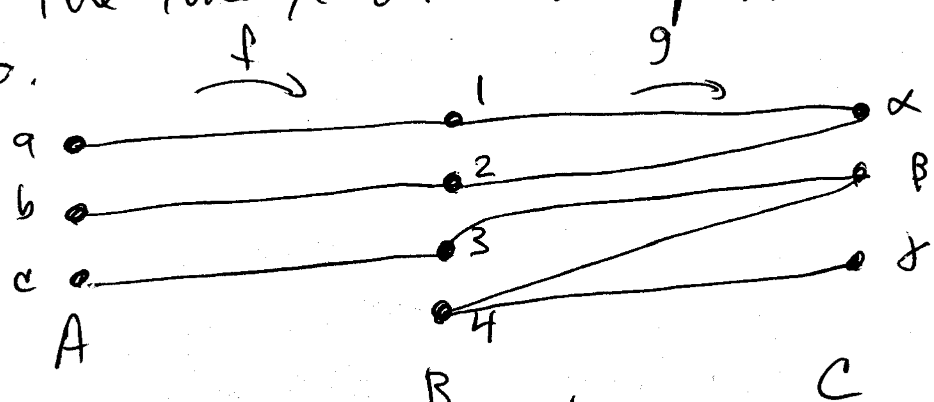
(b) $f = \{(a,1), (b,2), (c,3)\}$

Note that distinct points in A are mapped to distinct points in B so f is one-to-one. But no point in A is mapped to 4 so f is not onto. There are other correct answers

(c) $f = \{(1,a), (2,b), (3,c), (4,c)\}$

Since both 3 and 4 are mapped to c, f is not one-to-one, but all points in A are the image of some point in B so f is onto.

(d)



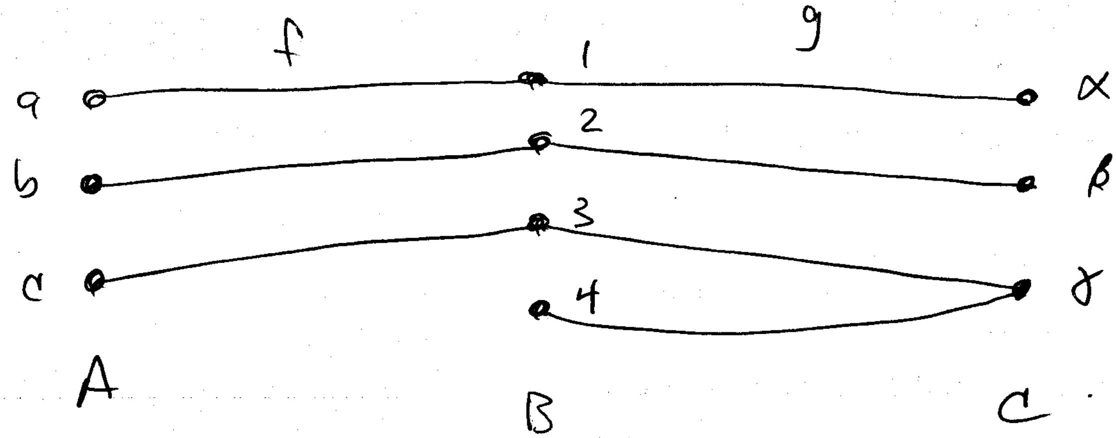
So $f = \{(a,1), (b,2), (c,3)\}$

$g = \{(1,\alpha), (2,\alpha), (3,\beta), (4,\beta), (4,\gamma)\}$

$g \circ f = \{(a,\alpha), (b,\alpha), (c,\beta)\}$

So $g \circ f$ is a function but g is not since $(4,\beta)$ and $(4,\gamma) \in g$ but $\beta \neq \gamma$.

(e)



$f = \{(a,1), (b,2), (c,3)\}$

$g = \{(1,\alpha), (2,\beta), (3,\gamma), (4,\gamma)\}$

$g \circ f = \{(a,\alpha), (b,\beta), (c,\gamma)\}$

g is not one-to-one since both 3 and 4 are mapped to γ.

3. (a) $f(1)=f(-1)=2$ so f is not one-to-one.

(b) Let $A=[0,\infty)$. To see that $f|_A$ is one-to-one, suppose that $x,y\in A$ satisfy $f(x)=f(y)$. This means $x^2+1=y^2+1$ or $x^2=y^2$ or that $|x|=|y|$. Since $x,y\in A$, $x,y\geq 0$ so $|x|=x$ and $|y|=y$. Hence $x=y$ and $f|_A$ is one-to-one.

(c) Let $B=[1,\infty)$. Since $B=\text{Rng}(f)$, f maps $\mathbb{R}$ onto B.

4. Suppose that $f:A\to B$ and $g:B\to C$ are one-to-one and suppose that $g\circ f(x)=g\circ f(y)$ for some $x,y\in A$. This means that $g(f(x))=g(f(y))$. Since $f(x),f(y)\in B$ and since g is one-to-one, $f(x)=f(y)$. Since $x,y\in A$ and f is one-to-one, $x=y$. Hence $g\circ f$ is one-to-one.