

MATH 290 – 15 JULY 2009 – EXAM 6

Answer all of the following questions on the answer sheets provided. Show all work, as partial credit may be given.

1. (5 pts.) Give an example of a graph of order 5 and size 6 that has at least one cycle (that is, a closed path) of length 3.

2. (4 pts. each) Let $A = \{a, b, c\}$, $B = \{1, 2, 3, 4\}$, and $C = \{\alpha, \beta, \gamma\}$. Give an example of each of the following. Your examples should be written as sets of ordered pairs.

(a) A relation f from A to B that is *not* a function.

(b) A function $f: A \rightarrow B$ that is one-to-one but not onto.

(c) A function $f: B \rightarrow A$ that is onto but not one-to-one.

(d) A function $f: A \rightarrow B$ and a relation g from B to C that is not a function, such that $g \circ f: A \rightarrow C$ is a function.

(e) A function $f: A \rightarrow B$ that is one-to-one and a function $g: B \rightarrow C$ that is not one-to-one, such that $g \circ f: A \rightarrow C$ is one-to-one.

3. (5 pts. each) Consider the function $f: \mathbf{R} \rightarrow \mathbf{R}$ given by $f(x) = x^2 + 1$.

(a) Give an example that shows that f is not one-to-one.

(b) Find a set $A \subseteq \mathbf{R}$ such that $f|_A$, the restriction of f to A , is one-to-one. You must provide a proof that the function $f|_A$ is one-to-one.

(c) Find a set $B \subseteq \mathbf{R}$ such that $f: \mathbf{R} \rightarrow B$ is onto.

4. (10 pts.) Prove that if $f: A \rightarrow B$ is one-to-one and $g: B \rightarrow C$ is one-to-one, then $g \circ f: A \rightarrow C$ is one-to-one.