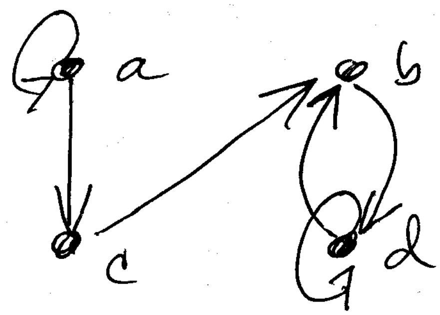
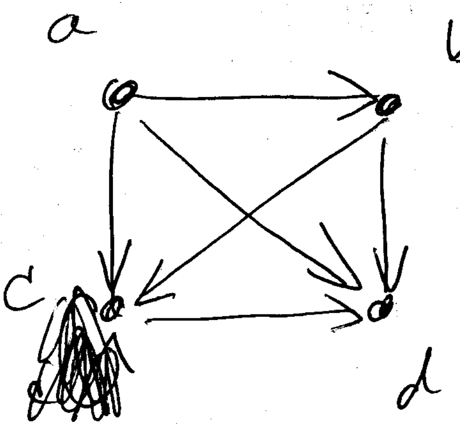
MATH 290 - EXAM 5 - SOLUTIONS

1. (a)



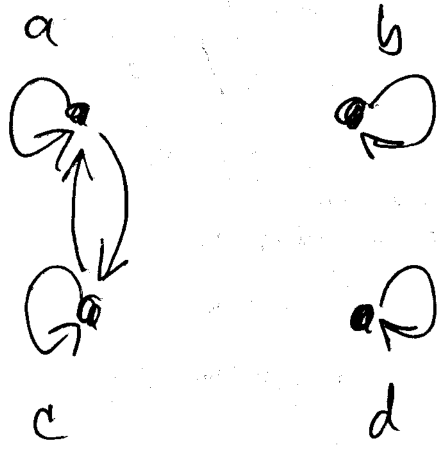
not reflexive
not symmetric
not anti-symmetric
not transitive

(b)

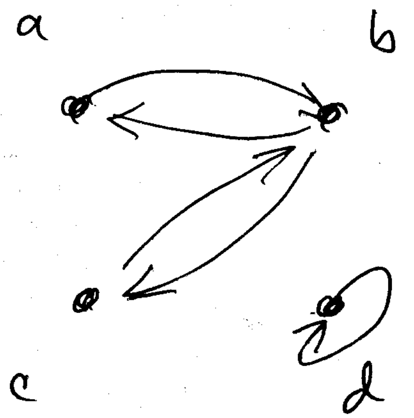


not reflexive
not symmetric
anti-symmetric
transitive

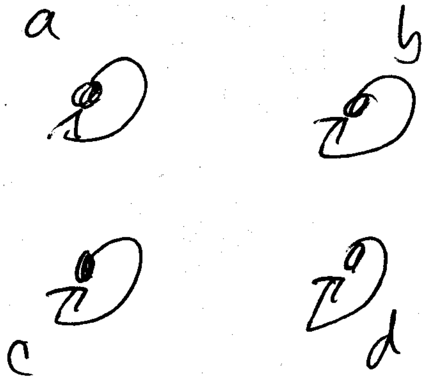
(c)



reflexive
symmetric
not anti-symmetric
transitive

(d) 

not reflexive
symmetric
not antisymmetric
not transitive

(e) 

reflexive
symmetric
anti-symmetric
transitive

2. (a) <u>Pf</u>: (1) Reflexive. Let $x \in \mathbb{R}$. Then $x - x = 0 \in \mathbb{Z}$.
Hence $x R x$ and R is reflexive.
(2) Symmetric. Let $x, y \in \mathbb{R}$ and suppose $x R y$.
Then $x - y \in \mathbb{Z}$. Then $y - x \in \mathbb{Z}$ so $y R x$, and R is symmetric
(3) Transitive. Let $x, y, z \in \mathbb{R}$ and suppose $x R y$ and $y R z$. This means $x - y \in \mathbb{Z}$ and $y - z \in \mathbb{Z}$. But then $x - z = (x-y) + (y-z) \in \mathbb{Z}$ so $x R z$, and R is transitive.

(b) $3/R = \{\dots, -1, 0, 1, 2, 3, 4, 5, \dots\} = \mathbb{Z}$

$\sqrt{2}/R = \{\dots, \sqrt{2}-1, \sqrt{2}, \sqrt{2}+1, \sqrt{2}+2, \dots\}$

$= \{\sqrt{2}+n : n \in \mathbb{Z}\}$.

3. (a) <u>Pf</u>: Suppose that $a, b \in \mathbb{N}$ and aRb and bRa. Then $b = ma$ for some $m \in \mathbb{N}$, $m \geq 1$ and $a = kb$ for some $k \in \mathbb{N}$, $k \geq 1$. Hence $a = kb = kma$ so that $km = 1$. But since $k, m \in \mathbb{N}$ it must be true that $k = m = 1$. Hence $a = b$ and R is anti-symmetric.

(b) Let $a = 5$ and $b = 7$. Then $b \neq ma$ for any $m \in \mathbb{N}$ and $a \neq kb$ for any $k \in \mathbb{N}$. so $a \not R b$ and $b \not R a$.

<u>NB</u>: Actually R is "divides" on $\mathbb{N}$.