MATH 290 – 8 JULY 2009 – EXAM 5

Answer all of the following questions on the answer sheets provided. Show all work, as partial credit may be given.

1. (4 pts. each) Indicate whether or not each of the given relations on the set $A = \{a,\ b,\ c,\ d\}$ is reflexive, symmetric, antisymmetric, or transitive. There is no need to justify your answer. (It might be helpful to draw a digraph representing each relation, but this is not required.)

(a) $R = \{(a,c),(a,a),(b,d),(c,b),(d,d),(d,b)\}$

(b) $R = \{(a,b),(b,c),(b,d),(a,d),(a,c),(c,d)\}$

(c) $R = \{(a,a),(b,b),(c,c),(d,d),(a,c),(c,a)\}$

(d) $R = \{(a,b),(b,a),(b,c),(c,b),(d,d)\}$

(e) $R = \{(a,a),(b,b),(c,c),(d,d)\}$

2. Define the relation R on the real numbers $\mathbf{R}$ as follows. For all $x,\ y \in \mathbf{R}$, xRy if and only if $x - y \in \mathbf{Z}$ (that is, $x - y$ is an integer).

(a) (10 pts.) Prove that R is an equivalence relation on $\mathbf{R}$.

(b) (5 pts.) Describe the equivalence class $3/R$ and the equivalence class $\sqrt{2}/R$.

3. Define the relation R on the natural numbers $\mathbf{N}$ as follows. For all $a,\ b \in \mathbf{N}$, aRb if and only if $b = ma$ for some natural number $m \geq 1$.

(a) (10 pts.) Prove that the relation R is anti-symmetric.

(b) (5 pts.) In fact, R is also reflexive and transitive, hence it is a partial order. Give an example that shows that R is not a linear order.