MATH 290 - B01

1.1 Propositions and Connectives

Deductive reasoning
Drawing correct conclusions from assumptions
Inductive reasoning
Postulating general principles based on observations
Laws of deductive reasoning based on a propositional calculus

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Definitions

① Proposition: A statement that is either true or false

e.g. $\sqrt{2}$ is irrational.
$1+1=5$
It is raining.
(NB: There can be no ambiguity in a proposition)
If it is raining then I will bring an umbrella.
This statement is false. (not a prop)
$x^2=36$

Designate propositions by P, Q, R, etc...

② Connectives

- conjunction (and) denoted $P \wedge Q$
- disjunction (or) " $P \vee Q$
- negation (not) " $\sim P$

Propositional form — a valid string of propositions linked by connectives

$P \wedge Q$ is true whenever both P and Q are true and false otherwise.

Truth table

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

③ Equivalent forms

Two forms are equivalent if they have same truth values e.g. #2 p.9

(h) $\sim(P \wedge Q)$ $\qquad$ $(\sim P) \vee (\sim Q)$

P	Q	$\sim(P \wedge Q)$
T	T	F
T	F	T
F	T	T
F	F	T

P	Q	$(\sim P) \vee (\sim Q)$
T	T	F
T	F	T
F	T	T
F	F	T

#3 (h) $(S \wedge R) \vee (S \wedge T)$

false

S false
T false
R true

#4 (h) $(P \vee Q) \vee R$ $\qquad$ $P \vee (Q \vee R)$

P	Q	R	$(P \vee Q) \vee R$	$P \vee (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

Forms are equivalent

#8 (e)

tautology - prop form whose value is always T

contradiction - prop form whose value is always F

$P \vee (\sim P)$ tautology

$P \wedge (\sim P)$ contradiction

$(Q \wedge \sim P) \wedge [\sim(P \wedge R)]$

P	Q	R	$(Q \wedge \sim P) \wedge (\sim(P \wedge R))$
T	T	T	F
T	T	F	F
T	F	T	F
T	F	F	F
F	T	T	T
F	T	F	⋮
F	F	T	
F	F	F	

④ Denial of a proposition

A proposition equivalent to $\sim S$ is a <u>denial</u> of S.

#10 (f) "$x<y$ or $m^2<1$" is P

Q is "$x<y$" R is "$m^2<1$"

$P = Q \vee R$

$\sim P = \sim(Q \vee R)$ = "It is not the case that $x<y$ or $m^2<1$."

$\sim P = (\sim Q) \wedge (\sim R)$ = "It is not the case that $x<y$ and it is not the case that $m^2<1$."

$\sim P$ = "$x \geq y$ and $m^2 \geq 1$"

(d) P = "641,371 is a composite integer"

$\sim P$ = "641,371 is prime."

(e) $\sim P$ = "roses ~~are~~ not red and violets are ~~not blue~~"

= "roses are not red or violets are not blue"

De Morgan's Laws

$\sim(P \wedge Q)$ equiv. to $\sim P \vee \sim Q$

$\sim(P \vee Q)$ equiv to $\sim P \wedge \sim Q$

$P \wedge (Q \vee R)$ equiv. to

$(P \wedge Q) \vee (P \wedge R)$

$P \vee (Q \wedge R)$ equiv to $(P \vee Q) \wedge (P \vee R)$

1.2 Conditionals + Biconditionals

① $P \Rightarrow Q$ "P implies Q"

"If P then Q"

If P is true then Q is true

If P is false I know nothing about Q

Truth table

P	Q	$P \Rightarrow Q$	$\sim P \vee Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

② <u>Converse</u> of $P \Rightarrow Q$ is $Q \Rightarrow P$

P	Q	$P \Rightarrow Q$	$Q \Rightarrow P$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

<u>Not equivalent.</u>

③ Contrapositive of $P \Rightarrow Q$ is

$\sim Q \Rightarrow \sim P$

$\sim P \vee Q \quad (P \Rightarrow Q)$

$\sim P \vee \sim(\sim Q)$

$\sim(\sim Q) \vee \sim P$

$\sim Q \Rightarrow \sim P$

P	Q	$P \Rightarrow Q$	$\sim Q \Rightarrow \sim P$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Equivalent!

④ $P \Leftrightarrow Q$ "P if and only if Q"

"P iff Q"

means $(P \Rightarrow Q) \wedge (Q \Rightarrow P)$

$P \Leftrightarrow Q$ holds only when P and Q always have same truth value.

P	Q	$P \Leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

#1 p.17 (f) (f integrable) $\Rightarrow$ (f bounded)

(h) (moon ~~full) $\Rightarrow$ (fish~~ will bite)

fish will bite $\Rightarrow$ moon is full

moon not full $\Rightarrow$ fish will not bite

(j) Olympic team $\Rightarrow$ 3 min 48 sec or less

$P \Rightarrow Q$ "P is sufficient for Q"

"Q is necessary for P"

("for P to hold it is necessary that Q also hold")

#2 (f) converse: f bounded $\Rightarrow$ f integrable

if f is bounded then it is integrable

contrapositive: f not bounded $\Rightarrow$ f not integrable

#4 (f) True (j) True

#5 (f) True (h) True

#8 (h) $6 \geq n-3$ only if $n>4$ or $n>10$

$$\cancel{(n>4) \vee (n>10) \Rightarrow (6 \geq n-3)}$$

$$(6 \geq n-3) \Rightarrow (n>4) \vee (n>10)$$

1.3 Quantifiers

$P =$ " $x^2 = 36$ "

can be true or false depending on x

we write $P(x)$

$Q =$ " S works in NYC "

write $Q(S)$

① The truth set of $P(x)$ is the set of all x for which $P(x)$ is true

To define such a set a universe of discourse is assumed.

For us, universe is usually

$\mathbb{N} = \{1, 2, \dots\}$

$\mathbb{Z} = \{\dots, -1, 0, 1, 2, \dots\}$

$\mathbb{Q}$ = rational numbers

$\mathbb{R}$ = real numbers

($\mathbb{C}$ = complex numbers)

e.g. $P(x) = "x^2 = 36"$

If universe is $\mathbb{N}$ then truth set of $P(x)$ is $\{6\}$

If univ. is $\mathbb{R}$ the truth set is $\{-6, 6\}$

② Existential quantifier

$(\exists x)\, P(x) \longrightarrow$ "there exists x (in the universe) such that P(x)"

$\longrightarrow$ means the truth set of P(x) is not empty.

e.g. $(\exists x)(x^2 = 36)$ is true (univ. is $\mathbb{R}$)

③ Universal quantifier.

$(\forall x)P(x)$ → "for all x (in the universe) $P(x)$"

→ means truth set of $P(x)$ is the entire universe

$(\forall x)(x^2=36)$ is false

(univ is $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ or $\mathbb{R}$)