MATH 214 – QUIZ 8 – SOLUTIONS

Use the method of undetermined coefficients to find a particular solution to the differential equation.

$$y'' + 2y' + y = 2e^{-t}.$$

(Hint: A fundamental system for the homogeneous equation is $\{e^{-t},\, t\,e^{-t}\}$.)

Solution: Since the right hand side of the equation is a solution to the homogeneous equation, and since t multiplied by it is also a solution, we look for a particular solution of the form $Y(t) = At^2\,e^{-t}$. We have

$$\begin{aligned} Y'(t) &= -At^2\,e^{-t} + 2At\,e^{-t} \\ Y''(t) &= At^2\,e^{-t} - 4At\,e^{-t} + 2A\,e^{-t}. \end{aligned}$$

Plugging into the original equation gives

$$At^2\,e^{-t} - 4At\,e^{-t} + 2A\,e^{-t} - 2At^2\,e^{-t} + 4At\,e^{-t} + At^2\,e^{-t} = 2e^{-t}.$$

Canceling terms gives

$$2A\,e^{-t} = 2e^{-t}$$

so that $A = 1$. Therefore, a particular solution is

$$Y(t) = t^2\,e^{-t}.$$

Use the method of undetermined coefficients to find a particular solution to the differential equation.

$$y'' + y' + y = t^2 + t.$$

(Hint: It will be enough to look for a particular solution in the form of a general quadratic polynomial.)

Solution: Following the hint, we look for a particular solution of the form $Y(t) = At^2 + Bt + C$. We have

$$\begin{aligned} Y'(t) &= 2At + B \\ Y''(t) &= 2A. \end{aligned}$$

Plugging into the original equation gives

$$2A + 2At + B + At^2 + Bt + C = t^2 + t.$$

Gathering like terms gives

$$At^2 + (2A + B)t + (2A + B + C) = t^2 + t$$

which leads to

$$\begin{aligned} A &= 1 \\ 2A + B &= 1 \\ 2A + B + C &= 0 \end{aligned}$$

The first equation gives $A = 1$, the second, $B = -1$, and the third, $C = -1$. Therefore, a particular solution is

$$Y(t) = t^2 - t - 1.$$

Use the method of undetermined coefficients to find a particular solution to the differential equation.

$$y'' + 2y' = 4\,\sin(2t).$$

Solution: We look for a particular solution of the form $Y(t) = A\,\cos(2t) + B\,\sin(2t)$. We have

$$\begin{aligned} Y'(t) &= -2A\,\sin(2t) + 2B\,\cos(2t) \\ Y''(t) &= -4A\,\cos(2t) - 4B\,\sin(2t). \end{aligned}$$

Plugging into the original equation gives

$$-4A\,\cos(2t) - 4B\,\sin(2t) - 4A\,\sin(2t) + 4B\,\cos(2t) = 4\,\sin(2t).$$

Gathering like terms gives

$$(-4A + 4B)\,\cos(2t) + (-4A - 4B)\,\sin(2t) = 4\,\sin(2t)$$

which leads to

$$\begin{aligned} -4A + 4B &= 0 \\ -4A - 4B &= 4 \end{aligned}$$

The first equation gives $A = B$, and the second, $-8B = 4$, so that $B = -1/2$ and $A = -1/2$. Therefore, a particular solution is

$$Y(t) = -\frac{1}{2}\,\cos(2t) - \frac{1}{2}\,\sin(2t).$$