MATH 214 – QUIZ 7 – SOLUTIONS

Use the method of reduction of order to find a second solution to the differential equation

$$t^2y'' - t(t+2)y' + (t+2)y = 0, \qquad t > 0, \qquad y_1(t) = t.$$

Solution: We seek a solution of the form $y_2(t) = v(t)\,t$. This gives us

$$\begin{aligned} y_2'(t) &= v(t) + v'(t)\,t \\ y_2''(t) &= 2v'(t) + v''(t)\,t. \end{aligned}$$

Plugging into the original equation gives

$$2t^2v'(t) + t^3v''(t) - t(t+2)(v(t) + tv'(t)) + (t+2)tv(t) = 0$$

which simplifies to

$$t^3v''(t) - t^3v'(t) = 0$$

or

$$v''(t) = v'(t).$$

A solution to this equation is $v'(t) = e^t$ and taking an antiderivative, we arrive at $v(t) = e^t$. Hence a second solution is $y_2(t) = te^t$.

Use the method of reduction of order to find a second solution to the differential equation

$$t^2y'' + 2ty' - 2y = 0, \qquad t > 0, \qquad y_1(t) = t.$$

Solution: We seek a solution of the form $y_2(t) = v(t)\,t$. This gives us

$$\begin{aligned} y_2'(t) &= v(t) + v'(t)\,t \\ y_2''(t) &= 2v'(t) + v''(t)\,t. \end{aligned}$$

Plugging into the original equation gives

$$2t^2v'(t) + t^3v''(t) + 2tv(t) + 2t^2v'(t) - 2tv(t) = 0$$

which simplifies to

$$t^3v''(t) + 4t^2v'(t) = 0$$

or

$$tv''(t) + 4v'(t) = 0.$$

Separating variables gives $v''/v' = -4/t$ which has a solution of $v' = t^{-4}$ and taking an antiderivative, we arrive at $v(t) = -(1/3)t^{-3}$. Hence a second solution is $y_2(t) = -(1/3)t^{-2}$.

Use the method of reduction of order to find a second solution to the differential equation

$$t^2y'' + 3ty' + y = 0, \qquad t > 0, \qquad y_1(t) = t^{-1}.$$

Solution: We seek a solution of the form $y_2(t) = v(t)\, t^{-1}$. This gives us

$$\begin{aligned} y_2'(t) &= -v(t)\, t^{-2} + v'(t)\, t^{-1} \\ y_2''(t) &= 2v(t)\, t^{-3} - 2v'(t)\, t^{-2} + v''(t)\, t^{-1}. \end{aligned}$$

Plugging into the original equation gives

$$tv''(t) - 2v'(t) + 2v(t)\, t^{-1} - 3v(t)\, t^{-1} + 3v'(t) + v(t)\, t^{-1} = 0$$

which simplifies to

$$tv''(t) + v'(t) = 0.$$

Separating variables gives $v''/v' = -1/t$ which has a solution of $v' = t^{-1}$ and taking an antiderivative, we arrive at $v(t) = \ln(t)$. Hence a second solution is $y_2(t) = t^{-1}\ln(t)$.