MATH 214 – QUIZ 6 – SOLUTIONS

Solve the initial value problem

$$y'' + 2y' + 2y = 0, \qquad y(0) = 2, \qquad y'(0) = -1.$$

Solution: The characteristic equation of this ODE is $r^2 + 2r + 2 = 0$, which has solutions $r_1 = -1 + i$, $r_2 = -1 - i$, and so the general solution is given by

$$y(t) = c_1\, e^{-t} \cos(t) + c_2\, e^{-t} \sin(t).$$

Plugging in the initial conditions gives the system of equations

$$\begin{aligned} 2 &= c_1 \\ -1 &= -c_1 + c_2 \end{aligned}$$

which leads to $c_1 = 2$, $c_2 = 1$ so that the final solution is

$$y(t) = 2\, e^{-t} \cos(t) + e^{-t} \sin(t).$$

Solve the initial value problem

$$y'' - 2y' + 6y = 0, \qquad y(0) = 1, \qquad y'(0) = -2.$$

Solution: The characteristic equation of this ODE is $r^2 - 2r + 6 = 0$, which has solutions $r_1 = 1 + i\sqrt{5}$, $r_2 = 1 - i\sqrt{5}$, and so the general solution is given by

$$y(t) = c_1\, e^{t} \cos(\sqrt{5}t) + c_2\, e^{t} \sin(\sqrt{5}t).$$

Plugging in the initial conditions gives the system of equations

$$\begin{aligned} 1 &= c_1 \\ -1 &= c_1 + \sqrt{5}c_2 \end{aligned}$$

which leads to $c_1 = 1$, $c_2 = -2/\sqrt{5}$ so that the final solution is

$$y(t) = e^{t} \cos(\sqrt{5}t) - \frac{2}{\sqrt{5}}\, e^{t} \sin(\sqrt{5}t).$$

Solve the initial value problem

$$4y'' + 9y = 0, \qquad y(\pi/3) = 1, \qquad y'(\pi/3) = -1.$$

Solution: The characteristic equation of this ODE is $4r^2 + 9 = 0$, which has solutions $r_1 = (3/2)i$, $r_2 = -(3/2)i$, and so the general solution is given by

$$y(t) = c_1 \cos(3t/2) + c_2 \sin(3t/2).$$

Plugging in the initial conditions gives the system of equations

$$\begin{aligned} 1 &= c_2 \\ -1 &= -c_1 \end{aligned}$$

which leads to $c_1 = 1$, $c_2 = 1$ so that the final solution is

$$y(t) = \cos(3t/2) + \sin(3t/2).$$