MATH 214 – QUIZ 5 – SOLUTIONS

Find the general solution to the equation

$$y'' + 4y' + 3y = 0$$

of the form

$$y(t) = c_1\, y_1(t) + c_2\, y_2(t),$$

and find the Wronskian of the solutions y_1 and y_2 that you found.

Solution: The characteristic equation of this ODE is $r^2 + 4r + 3 = 0$, which factors to $(r+3)(r+1) = 0$ so that the roots are $r_1 = -3$ and $r_2 = -1$. Hence two solutions to the equation are $y_1(t) = e^{-3t}$ and $y_2(t) = e^{-t}$ and the general solution can be written

$$y(t) = c_1\, e^{-3t} + c_2\, e^{-t}.$$

The Wronskian of y_1 and y_2 is

$$W(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{-3t}(-e^{-t}) - e^{-t}(-3e^{-3t}) = 2\, e^{-4t}.$$

Find the general solution to the equation

$$y'' + 8y' - 9y = 0$$

of the form

$$y(t) = c_1\, y_1(t) + c_2\, y_2(t),$$

and find the Wronskian of the solutions y_1 and y_2 that you found.

Solution: The characteristic equation of this ODE is $r^2 + 8r - 9 = 0$, which factors to $(r+9)(r-1) = 0$ so that the roots are $r_1 = -9$ and $r_2 = 1$. Hence two solutions to the equation are $y_1(t) = e^{-9t}$ and $y_2(t) = e^{t}$ and the general solution can be written

$$y(t) = c_1\, e^{-9t} + c_2\, e^{t}.$$

The Wronskian of y_1 and y_2 is

$$W(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{-9t} e^{t} - e^{t}(-9e^{-9t}) = 10\, e^{-8t}.$$

Find the general solution to the equation

$$2y'' - 3y' + y = 0$$

of the form

$$y(t) = c_1\, y_1(t) + c_2\, y_2(t),$$

and find the Wronskian of the solutions y_1 and y_2 that you found.

Solution: The characteristic equation of this ODE is $2r^2 - 3r + 1 = 0$, which factors to $(2r-1)(r-1) = 0$ so that the roots are $r_1 = 1/2$ and $r_2 = 1$. Hence two solutions to the equation are $y_1(t) = e^{t/2}$ and $y_2(t) = e^t$ and the general solution can be written

$$y(t) = c_1\, e^{t/2} + c_2\, e^t.$$

The Wronskian of y_1 and y_2 is

$$W(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{t/2} e^t - e^t \frac{1}{2} e^{t/2} = \frac{1}{2}\, e^{3t/2}.$$