MATH 214 – QUIZ 3 – SOLUTIONS

Solve the following initial value problem: $y' + 3y = 1 + e^{-2t}$, $y(0) = 1$.

Solution: The integrating factor in this case is $\mu(t) = e^{\int 3dt} = e^{3t}$ which leads to the following solution.

$$\begin{aligned} e^{3t}y' + 3e^{3t}y &= e^{3t}(1 + e^{-2t}) \\ \frac{d}{dt}(e^{3t}y) &= e^{3t} + e^{t} \\ e^{3t}y &= \frac{1}{3}e^{3t} + e^{t} + c \\ y &= \frac{1}{3} + e^{-2t} + c\,e^{-3t} \end{aligned}$$

$y(0) = 1$ leads to $1 = 1/3 + 1 + c$ or $c = 1/3$. Hence the final solution is

$$y = \frac{1}{3} + e^{-2t} + \frac{1}{3}\,e^{-3t}.$$

Solve the following initial value problem: $y' - 2y = t^2e^{2t}$, $y(0) = 1$.

Solution: The integrating factor in this case is $\mu(t) = e^{\int -2dt} = e^{-2t}$ which leads to the following solution.

$$\begin{aligned} e^{-2t}y' - 2e^{-2t}y &= e^{-2t}(t^2e^{2t}) \\ \frac{d}{dt}(e^{-2t}y) &= t^2 \\ e^{-2t}y &= \frac{1}{3}t^3 + c \\ y &= \frac{1}{3}e^{2t}\,t^3 + c\,e^{2t} \end{aligned}$$

$y(0) = 1$ leads to $1 = 0 + c$ or $c = 1$. Hence the final solution is

$$y = \frac{1}{3}e^{2t}\,t^3 + e^{2t}.$$

Solve the following initial value problem: $y' + y = te^{-t} + 1$, $y(0) = 2$.

Solution: The integrating factor in this case is $\mu(t) = e^{\int dt} = e^t$ which leads to the following solution.

$$\begin{aligned} e^t y' + e^t y &= e^t(te^{-t} + 1) \\ \frac{d}{dt}(e^t y) &= t + e^t \\ e^t y &= \frac{1}{2}t^2 + e^t + c \\ y &= \frac{1}{2}e^{-t}\,t^2 + 1 + c\,e^{-t} \end{aligned}$$

$y(0) = 2$ leads to $2 = 0 + 1 + c$ or $c = 1$. Hence the final solution is

$$y = \frac{1}{2}e^{-t}\,t^2 + 1 + e^{-t}.$$