

MATH 214 – QUIZ 1 – SOLUTIONS

Find all equilibrium solutions to the differential equation $y' = y(y - 3)$ and indicate whether they are attracting or repelling. (Hint: It may be helpful for you to sketch a direction field for the equation, but it is not required.)

Solution: Equilibrium solutions mean that $y' = 0$. This occurs when $y = 0$ and when $y = 3$. If $y < 0$ then $y' = y(y - 3) > 0$ which means that solutions passing through points below the line $y = 0$ have positive slope and hence move toward $y = 0$. If $0 < y < 3$ then $y' = y(y - 3) < 0$ which means that solutions passing through points above the line $y = 0$ and below the line $y = 3$ have negative slope and hence move toward $y = 0$ and away from $y = 3$. If $y > 3$ then $y' = y(y - 3) > 0$ which means that solutions passing through points above the line $y = 0$ have positive slope and hence move away from $y = 3$. This means that $y = 0$ is *attracting*, and $y = 3$ is *repelling*.

Find all equilibrium solutions to the differential equation $y' = y(3 - y)$ and indicate whether they are attracting or repelling. (Hint: It may be helpful for you to sketch a direction field for the equation, but it is not required.)

Solution: Equilibrium solutions mean that $y' = 0$. This occurs when $y = 0$ and when $y = 3$. If $y < 0$ then $y' = y(3 - y) < 0$ which means that solutions passing through points below the line $y = 0$ have negative slope and hence move away from $y = 0$. If $0 < y < 3$ then $y' = y(3 - y) > 0$ which means that solutions passing through points above the line $y = 0$ and below the line $y = 3$ have positive slope and hence move away from $y = 0$ and toward $y = 3$. If $y > 3$ then $y' = y(3 - y) < 0$ which means that solutions passing through points above the line $y = 0$ have negative slope and hence move toward $y = 3$. This means that $y = 0$ is *repelling*, and $y = 3$ is *attracting*.

Find all equilibrium solutions to the differential equation $y' = y(y + 3)$ and indicate whether they are attracting or repelling. (Hint: It may be helpful for you to sketch a direction field for the equation, but it is not required.)

Solution: Equilibrium solutions mean that $y' = 0$. This occurs when $y = 0$ and when $y = -3$. If $y < -3$ then $y' = y(y + 3) > 0$ which means that solutions passing through points below the line $y = -3$ have positive slope and hence move toward $y = -3$. If $-3 < y < 0$ then $y' = y(y + 3) < 0$ which means that solutions passing through points above the line $y = -3$ and below the line $y = 0$ have negative slope and hence move toward $y = -3$ and away from $y = 0$. If $y > 0$ then $y' = y(y + 3) > 0$ which means that solutions passing through points above the line $y = 0$ have positive slope and hence move away from $y = 0$. This means that $y = -3$ is *attracting*, and $y = 0$ is *repelling*.