Answer all of the following questions on the answer sheets provided. Show all work, as partial credit may be given. This exam is counted out of a total of 40 points.

1. (5 pts. each) Consider the initial value problem $y' = y^2(y - 2)$, $y(0) = y_0$.

   (a) Find all equilibrium solutions to this equation. Do not attempt to solve the IVP.

   (b) Describe the long term behavior (that is, the behavior as $t \to \infty$) of the solutions to the IVP for various values of y_0. Give as complete a description as possible. You may sketch a direction field to help you solve this problem but it is not necessary.

2. (5 pts.) A bathtub initially contains 30 gallons of clean water. Salt water with a concentration of 20 grams of salt per gallon is poured into the tub at a rate of 4 gallons per minute and the drain is opened to drain the tub at the same rate. Set up and solve an initial value problem giving $Q(t)$, the amount of salt in the tub at time t.

3. (5 pts.) Suppose that the field mouse population, $p(t)$, in a certain field satisfies the differential equation $\dfrac{dp}{dt} = p - 800$, where t is measured in years. If the initial population $p(0) = 600$, solve the initial value problem and find the time T at which the population becomes extinct.

4. (5 pts. each) Solve each of the following problems.

   (a) $y \dfrac{dy}{dt} = t^2$, $y(0) = 2$.

   (b) $\dfrac{dy}{dt} - 3y = 6e^t$, $y(0) = -2$.

   (c) $(3x^2 + 2xy) + (2y + x^2)\, y' = 0$. (Hint: This equation is exact.)

5. (5 pts.) Find an interval of t on which the solution to the initial value problem $(4 - t)y' + 2ty = 3t^2$, $y(-3) = 1$ is certain to exist. Do not solve the IVP!