MATH 214 – 27 SEPTEMBER 2011 – EXAM 1

Answer all of the following questions on the answer sheets provided. Show all work, as partial credit may be given. This exam is counted out of a total of 40 points.

1. (5 pts. each) Consider the initial value problem $y' = y(y-2)^2$, $y(0) = y_0$.

   (a) Find all equilibrium solutions to this equation. Do not attempt to solve the IVP.

   (b) Describe the long term behavior (that is, the behavior as $t \to \infty$) of the solutions to the IVP for various values of y_0. Give as complete a description as possible. You may sketch a direction field to help you solve this problem but it is not necessary.

2. (5 pts.) 20 grams of an undesirable chemical has been introduced into a bathtub containing 30 gallons of water. You turn on the spigot and allow uncontaminated water to come in at a rate of 4 gallons per minute and open the drain to let the water out at the same rate. Set up and solve an initial value problem giving $Q(t)$, the amount of chemical in the tub at time t.

3. (5 pts.) Suppose that a person takes out a loan at an *annual* interest rate is 6%, and on which he pays \$100 *per month*. Suppose that the interest rate is compounded continuously and that the payments are made continuously. Set up and solve an initial value problem giving the remaining balance on the loan $A(t)$ if the initial amount is $A(0) = A_0$.

4. (5 pts. each) Solve each of the following problems.

   (a) $\dfrac{dy}{dt} - 2y = 4$, $y(0) = 2$.

   (b) $\dfrac{dy}{dt} - 2y = 4e^t$, $y(0) = -2$.

   (c) $(3x^2 - 2xy) + (y - x^2)\, y' = 0$. (Hint: This equation is exact.)

5. (5 pts.) Find an interval of t on which the solution to the initial value problem $(4-t)y' + 2ty = 3t^2$, $y(-3) = 1$ is certain to exist. Do not solve the IVP!