MATH 213 – QUIZ 13 – 1 MAY 2012

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Compute the scalar line integral $\int_C ((x-y)\,ds$ *where* C *is the upper half of the circle of radius* 2 *centered at the origin and is given by* $C\colon \mathbf{r}(t) = \langle 2\cos t,\, 2\sin t\rangle,\ 0 \le t \le \pi$.

$\vec{r}(t) = \langle 2\cos t, 2\sin t\rangle$

$\vec{r}\,'(t) = \langle -2\sin t, 2\cos t\rangle$

$|\vec{r}\,'(t)| = (4\sin^2 t + 4\cos^2 t)^{1/2} = 2$

$$\int_C (x-y)\,ds = \int_0^\pi (2\cos t - 2\sin t)(2)\,dt$$

$$= 4\int_0^\pi \cos t - \sin t\,dt = 4(\sin t + \cos t)\Big|_0^\pi$$

$$= 4(\sin\pi - \sin 0 + \cos\pi - \cos 0)$$

$$= 4(0 - 0 - 1 - 1) = -8$$

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Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Given that the vector field $\mathbf{F} = \langle y, x, 1\rangle$ *is conservative, find a function* $\varphi(x, y, z)$ *such that* $\mathbf{F} = \nabla\varphi$. *Then evaluate* $\int_C \mathbf{F}\cdot d\mathbf{r}$ *where* C *is the circle of radius 4 centered at the origin. (Hint: This last part is easier than it looks!)*

$\varphi_x = y \longrightarrow \varphi = xy + g(y,z)$

$\varphi_y = x \qquad \varphi_y = x + g_y(y,z)$

$\varphi_z = 1$

$x = x + g_y(y,z) \longrightarrow g_y = 0$

$\therefore g(y,z) = h(z)$

$\therefore \varphi = xy + h(z) \longrightarrow \varphi_z = h'(z)$.

$\therefore h'(z) = 1 \longrightarrow h(z) = z + c$.

$\therefore \varphi(x,y,z) = xy + z + c$

So $\varphi(x,y,z) = xy + z$ is a solution //

Since $\vec{F}$ is conservative, its line integral around any closed curve is zero.

$\therefore \int_C \vec{F}\cdot d\vec{r} = 0$ //

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Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Given that the vector field $\mathbf{F} = \langle -y,\, y - x\rangle$ *is conservative, find a function* $\varphi(x, y)$ *such that* $\mathbf{F} = \nabla\varphi$. *Then evaluate* $\int_C \mathbf{F}\cdot d\mathbf{r}$ *where* C *is the curve given by* $C\colon \mathbf{r}(t) = \langle t,\, 2t^2\rangle$, $1 \le t \le 2$. *(Hint: This last part is easier than it looks!)*

$\varphi_x = -y \longrightarrow \varphi = -xy + g(y)$

$\varphi_y = y - x \qquad \varphi_y = -x + g'(y).$

$y - x = -x + g'(y) \rightarrow g'(y) = y$

$g(y) = \frac{1}{2}y^2 + c.$

$\therefore \varphi = -xy + \frac{1}{2}y^2 + c$

Taking $c = 0$ gives $\varphi = -xy + \frac{1}{2}y^2$ as a solution. //

By the Fundamental Theorem for line integrals,

$$\int_C \vec{F}\cdot d\vec{r} = \varphi(\vec{r}(2)) - \varphi(\vec{r}(1))$$
$$= \varphi(2, 8) - \varphi(1, 2)$$
$$= (-16 + 32) - (-2 + 2) = 16 //.$$