MATH 213 – QUIZ 11 – 17 APRIL 2012

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Evaluate the triple integral $\int_1^6 \int_0^{4-2y} \int_0^{4-2y-x} \frac{1}{y}\, dz\, dx\, dy.$

$$\int_1^6 \int_0^{4-2y} \int_0^{4-2y-x} \frac{1}{y}\, dz\, dx\, dy$$

$$= \int_1^6 \int_0^{4-2y} \left(\frac{z}{y} \Big|_0^{4-2y-x} \right) dx\, dy$$

$$= \int_1^6 \int_0^{4-2y} \left(\frac{4}{y} - 2 - \frac{x}{y} \right) dx\, dy$$

$$= \int_1^6 \frac{4x}{y} - 2x - \frac{1}{2}\frac{x^2}{y} \Big|_0^{4-2y} dy$$

$$= \int_1^6 \frac{16-8y}{y} - (8-4y) - \frac{(16-16y+4y^2)}{2y}\, dy$$

$$= \int_1^6 \frac{16}{y} - 8 - 8 + 4y - \frac{8}{y} + 8 - 2y\, dy$$

$$= \int_1^6 \frac{8}{y} + 2y - 8\, dy = 8\ln|y| + y^2 - 8y \Big|_1^6$$

$$= 8\ln(6) + 36 - 48 - (0 + 1 - 8) = \boxed{8\ln 6 - 5}$$

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Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Evaluate the integral $\iint_R (x^2 + y^2)\, dA$ *in polar coordinates where* $R = \{(x, y) : x^2 + y^2 \leq 9, y \geq 0\}$.

$$\iint_R (x^2+y^2)\,dA = \int_0^{\pi}\int_0^{3} r^2 \cdot r\,dr\,d\theta$$

$$= \int_0^{\pi}\int_0^{3} r^3\,dr\,d\theta = \int_0^{\pi}\left(\frac{1}{4}r^4\Big|_0^3\right)d\theta$$

$$= \frac{81}{4}\int_0^{\pi} d\theta = \frac{81\pi}{4}$$

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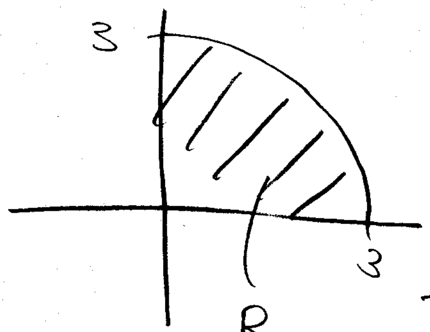
Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Evaluate the integral $\iint_R 2xy\,dA$ *in polar coordinates where* $R = \{(r, \theta) : 0 \le r \le 3, 0 \le \theta \le \pi/2\}$. *(Hint: Remember the double angle formula* $2\sin(x)\cos(x) = \sin(2x)$.*)*



$$\iint_R 2xy\,dA = \int_0^{\pi/2}\int_0^3 2r\cos\theta \cdot r\sin\theta\, r\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^3 r^3\, 2\sin\theta\cos\theta\,dr\,d\theta$$

$$= \int_0^{\pi/2}\int_0^3 r^3 \sin(2\theta)\,dr\,d\theta = \int_0^{\pi/2} \left(\frac{1}{4}r^4\Big|_0^3\right)\sin 2\theta\,d\theta$$

$$= \int_0^{\pi/2} \frac{81}{4}\sin 2\theta\,d\theta = -\frac{81}{8}\cos 2\theta\Big|_0^{\pi/2} = \frac{81}{8} - \left(-\frac{81}{8}\right)$$

$$= \frac{81}{4} \;//$$