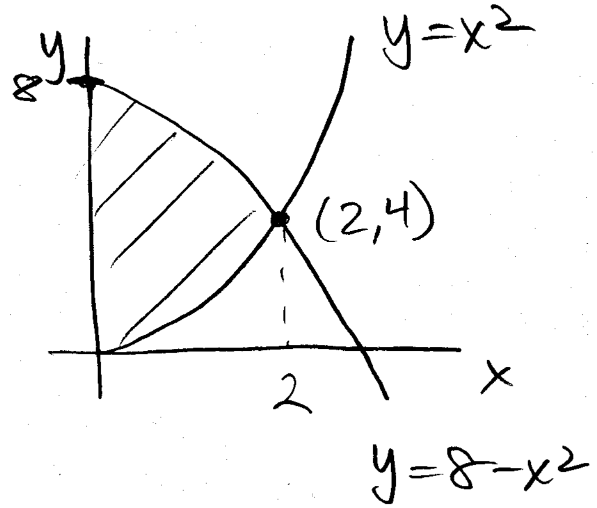
MATH 213 – QUIZ 10 – 10 APRIL 2012

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Evaluate $\iint_R (x+y)\,dA$ *where R is the region in the first quadrant bounded by $y = x^2$, and $y = 8 - x^2$. (Hint: Sketch the region first.)*



$$\iint_R (x+y)\,dA$$

$$= \int_0^2 \int_{x^2}^{8-x^2} x+y \; dy\,dx$$

$$= \int_0^2 xy + \tfrac{1}{2}y^2 \Big|_{x^2}^{8-x^2} dx$$

$$= \int_0^2 x(8-x^2) + \tfrac{1}{2}\underbrace{(8-x^2)^2}_{64-16x^2+x^4} - x^3 - \tfrac{1}{2}x^4 \, dx$$

$$= \int_0^2 8x - x^3 + 32 - 8x^2 + \cancel{\tfrac{1}{2}x^4} - x^3 - \cancel{\tfrac{1}{2}x^4} \; dx$$

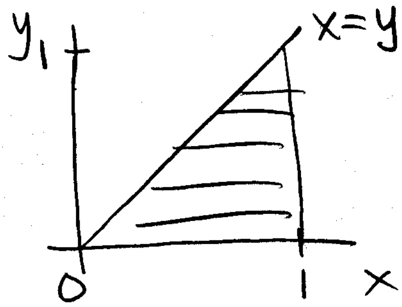
$$= \int_0^2 -2x^3 - 8x^2 + 8x + 32 \, dx$$

$$= -\tfrac{1}{2}x^4 - \tfrac{8}{3}x^3 + 4x^2 + 32x \Big|_0^2$$

$$= -8 - \tfrac{64}{3} + 16 + 64 = 72 - \tfrac{64}{3} = \tfrac{152}{3} \; //$$

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Reverse the order of integration in this integral and evaluate: $\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$. *(Hint: Sketch the region first.)*



$$\int_0^1 \int_y^1 e^{x^2}\, dx\, dy$$

$$= \int_0^1 \int_0^x e^{x^2}\, dy\, dx$$

$$= \int_0^1 y e^{x^2} \Big|_0^x \, dx$$

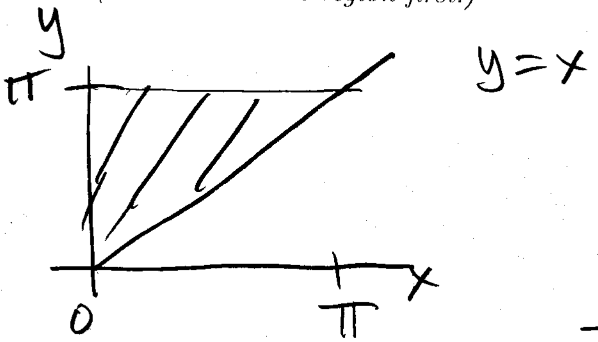
$$= \int_0^1 x e^{x^2}\, dx$$

$u = x^2$
$du = 2x\, dx$
$x = 0 \rightarrow u = 0$
$x = 1 \rightarrow u = 1$

$$= \int_0^1 \frac{1}{2} e^u\, du = \frac{1}{2} e^u \Big|_0^1 = \frac{1}{2}(e-1) \,//.$$

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Reverse the order of integration in the following integral and evaluate: $\int_0^{\pi}\int_x^{\pi} \sin(y^2)\,dy\,dx$. *(Hint: Sketch the region first.)*



$$\int_0^{\pi}\int_x^{\pi} \sin(y^2)\,dy\,dx$$

$$= \int_0^{\pi}\int_0^{y} \sin(y^2)\,dx\,dy$$

$$= \int_0^{\pi} x\sin(y^2)\Big|_0^y\,dy = \int_0^{\pi} y\sin(y^2)\,dy$$

$u = y^2$
$du = 2y$
$y=0 \Rightarrow u=0$
$y=\pi \Rightarrow u=\pi^2$

$$= \int_0^{\pi^2} \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u)\Big|_0^{\pi^2}$$

$$= \frac{1}{2}\left(1-\cos(\pi^2)\right)$$