MATH 213 – QUIZ 8 – 27 MARCH 2012

Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Find the maximum rate of change of the function $g(x, y) = x^2 + 2xy - x^2y - y^2$ *at the point* $(1, 2)$. *Also find a unit vector giving the direction of the maximum rate of change at this point.*

$$\nabla g = \langle 2x + 2y - 2xy,\ 2x - x^2 - 2y \rangle$$

$$\nabla g(1,2) = \langle 2, -3 \rangle$$

max rate of change at $(1,2)$ $= |\nabla g(1,2)| = \sqrt{13}$ //

direction of max rate of change at $(1,2)$ $= \dfrac{\nabla g(1,2)}{|\nabla g(1,2)|} = \left\langle \dfrac{2}{\sqrt{13}}, \dfrac{-3}{\sqrt{13}} \right\rangle$ //

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Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Use the chain rule to compute w_r, w_s, and w_t if $w = (2x+y^2)^3$, $x = \frac{r}{st}$, $y = rst$.

$$\frac{\partial w}{\partial x} = 3(2x+y^2)^2(2) = 6(2x+y^2)^2$$

$$\frac{\partial w}{\partial y} = 3(2x+y^2)^2(2y) = 6y(2x+y^2)^2$$

$$\frac{\partial x}{\partial r} = \frac{1}{st} \quad \frac{\partial x}{\partial s} = \frac{-r}{s^2t} \quad \frac{\partial x}{\partial t} = \frac{-r}{st^2}$$

$$\frac{\partial y}{\partial r} = st \quad \frac{\partial y}{\partial s} = rt \quad \frac{\partial y}{\partial t} = rs.$$

$$w_r = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} = 6(2x+y^2)^2\left[\frac{1}{st} + yst\right]$$

$$= 6\left(\frac{2r}{st} + r^2s^2t^2\right)^2\left[\frac{1}{st} + rs^2t^2\right]. \,/\!/$$

$$w_s = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} = 6(2x+y^2)^2\left[\frac{-r}{s^2t} + yrt\right]$$

$$= 6\left(\frac{2r}{st} + r^2s^2t^2\right)^2\left[\frac{-r}{s^2t} + r^2st^2\right] \,/\!/$$

$$w_t = \frac{\partial w}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial t} = 6(2x+y^2)^2\left[\frac{-r}{st^2} + yrs\right]$$

$$= 6\left(\frac{2r}{st} + r^2s^2t^2\right)^2\left[\frac{-r}{st^2} + r^2s^2t\right]. \,/\!/$$

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Answer the following question in the space provided. There is no need to justify your answers. This quiz is worth 5 points.

Compute the directional derivative of $f(x,y) = \dfrac{x}{x-y}$ *at the point* $(4,1)$ *and in the direction of the vector* $\langle -1, 2\rangle$.

$$\vec{u} = \frac{\langle -1,2\rangle}{\|\langle -1,2\rangle\|} = \frac{\langle -1,2\rangle}{(1^2+2^2)^{1/2}} = \left\langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$\nabla f = \left\langle \frac{(x-y)-x}{(x-y)^2}, \frac{x}{(x-y)^2} \right\rangle = \left\langle \frac{-y}{(x-y)^2}, \frac{x}{(x-y)^2} \right\rangle$$

$$\nabla f(4,1) \cdot \vec{u} = \left\langle \frac{-1}{9}, \frac{4}{9} \right\rangle \cdot \left\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \right\rangle$$

$$= \frac{9}{9\sqrt{5}} = \frac{1}{\sqrt{5}} \;//$$