MATH 213 – QUIZ 4 – 21 FEBRUARY 2012

Answer the following question in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit. This quiz is worth 5 points.

Let $\mathbf{r}(t) = \langle t\,\sin(t), t\,\cos(t), 2t\rangle$, $t \geq 0$. *Compute the arclength function* $s(t) = \int_0^t |r'(u)|\,du$.

$$\vec{r}\,'(t) = \langle t\cos t + \sin t, -t\sin t + \cos t, 2\rangle$$

$$|\vec{r}\,'(t)| = \left((t\cos t + \sin t)^2 + (-t\sin t + \cos t)^2 + 4\right)^{1/2}$$

$$= \left(t^2\cos^2 t + \cancel{2t\cos t\sin t} + \sin^2 t + t^2\sin^2 t \cancel{- 2t\sin t\cos t} + \cos^2 t + 4\right)^{1/2}$$

$$= \left(t^2(\cos^2 t + \sin^2 t) + (\sin^2 t + \cos^2 t) + 4\right)^{1/2}$$

$$= (t^2+5)^{1/2} \; //$$

$$\therefore\; s(t) = \int_0^t (u^2+5)^{1/2}\,du \; //$$

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Let $\mathbf{r}(t) = 2t\,\mathbf{i} + (t-6)\,\mathbf{j} - 3t\,\mathbf{k}$, $t \geq 0$. *Find a parametrization* $\mathbf{R}(s)$ *that traces out the same trajectory as* $r(t)$ *but for which* $|\mathbf{R}'(s)| = 1$. *(This is the arclength parametrization of* $\mathbf{r}$*).*

$\vec{r}'(t) = 2\vec{i} + \vec{j} - 3\vec{k}$

$|\vec{r}'(t)| = (4+1+9)^{1/2} = \sqrt{14}$

$s(t) = \int_0^t \sqrt{14}\, u\, du = \sqrt{14}\, t$

Let $s = \sqrt{14}\,t$. Then $t = \frac{s}{\sqrt{14}}$ and

$\vec{R}(s) = \frac{2}{\sqrt{14}} s\, \vec{i} + \left(\frac{s}{\sqrt{14}} - 6\right)\vec{j} - \frac{3}{\sqrt{14}} s\, \vec{k}$ //

It is easy to verify that $|\vec{R}'(s)| = 1$.

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Let $\mathbf{r}(t) = \langle \sin(2t), \cos(2t), 2t\rangle$, $t \geq 0$. *Compute the curvature* $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$ *for this curve, where* $\mathbf{T}(t)$ *is the unit tangent vector for* $\mathbf{r}(t)$.

$$\vec{r}'(t) = \langle 2\cos 2t, -2\sin 2t, 2\rangle$$

$$|\vec{r}'(t)| = (4\cos^2 2t + 4\sin^2 2t + 4)^{1/2}$$
$$= \sqrt{8} = 2\sqrt{2}.$$

$$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \left\langle \frac{1}{\sqrt{2}}\cos 2t, -\frac{1}{\sqrt{2}}\sin 2t, \frac{1}{\sqrt{2}} \right\rangle$$

$$\vec{T}'(t) = \left\langle -\frac{2}{\sqrt{2}}\sin 2t, -\frac{2}{\sqrt{2}}\cos 2t, 0 \right\rangle$$

$$|\vec{T}'(t)| = (2\sin^2 2t + 2\cos^2 2t)^{1/2} = \sqrt{2}.$$

$$\therefore \; \kappa(t) = \frac{\sqrt{2}}{2\sqrt{2}} = \frac{1}{2} \; //$$