MATH 213 – QUIZ 2 – 7 FEBRUARY 2012

Answer the following question in the space provided. Show all work as partial credit may be given. Answers without justification, even if they are correct, will earn no credit. This quiz is worth 5 points.

Let $\mathbf{u} = \langle -3, 0, 2\rangle$ *and* $\mathbf{v} = \langle -3, 5, 1\rangle$. *Find* $proj_{\mathbf{v}}(\mathbf{u})$ *and* $proj_{\mathbf{u}}(\mathbf{v})$.

$$\vec{u}\cdot\vec{v} = \langle -3,0,2\rangle \cdot \langle -3,5,1\rangle = 9+0+2 = 11$$

$$|\vec{u}| = (9+0+4)^{1/2} = \sqrt{13} \quad |\vec{v}| = (9+25+1)^{1/2} = \sqrt{35}$$

$$proj_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\right)\vec{v} = \frac{11}{35}\langle -3,5,1\rangle$$

$$= \left\langle \frac{-33}{35}, \frac{55}{35}, \frac{11}{35}\right\rangle //$$

$$proj_{\vec{u}}(\vec{v}) = \left(\frac{\vec{v}\cdot\vec{u}}{|\vec{u}|^2}\right)\vec{u} = \frac{11}{13}\langle -3,0,2\rangle$$

$$= \left\langle \frac{-33}{13}, 0, \frac{22}{13}\right\rangle //$$

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Let $\mathbf{u} = 2\mathbf{i} - 3\mathbf{k}$ *and* $\mathbf{v} = \mathbf{i} - 4\mathbf{j} - 6\mathbf{k}$. *Write* $\mathbf{u}$ *as the sum of a vector parallel to* $\mathbf{v}$ *and a vector perpendicular to* $\mathbf{v}$.

$$\vec{u} = \underbrace{\mathrm{proj}_{\vec{v}}(\vec{u})}_{\text{parallel to } \vec{v}} + \underbrace{\left(\vec{u} - \mathrm{proj}_{\vec{v}}(\vec{u})\right)}_{\text{perpendicular to } \vec{v}}$$

$$\vec{u} \cdot \vec{v} = 2 + 0 + 18 = 20$$

$$|\vec{v}|^2 = (1 + 16 + 36) = 53$$

$$\mathrm{proj}_{\vec{v}}(\vec{u}) = \left(\frac{\vec{u} \cdot \vec{v}}{|\vec{v}|^2}\right)\vec{v} = \frac{20}{53}\left(\vec{i} - 4\vec{j} - 6\vec{k}\right)$$

$$= \frac{20}{53}\vec{i} - \frac{80}{53}\vec{j} - \frac{120}{53}\vec{k}$$

$$\vec{u} - \mathrm{proj}_{\vec{v}}(\vec{u}) = \left(2\vec{i} - 3\vec{k}\right) - \left(\frac{20}{53}\vec{i} - \frac{80}{53}\vec{j} - \frac{120}{53}\vec{k}\right)$$

$$= \frac{86}{53}\vec{i} + \frac{80}{53}\vec{j} - \frac{39}{53}\vec{k}$$

$$\vec{u} = \underbrace{\left(\frac{20}{53}\vec{i} - \frac{80}{53}\vec{j} - \frac{120}{53}\vec{k}\right)}_{\text{parallel to } \vec{v}} + \underbrace{\left(\frac{86}{53}\vec{i} + \frac{80}{53}\vec{j} - \frac{39}{53}\vec{k}\right)}_{\text{perpendicular to } \vec{v}} \quad //$$

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Let $\mathbf{u} = 2\mathbf{j} - 3\mathbf{k}$ *and* $\mathbf{v} = \mathbf{i} - 6\mathbf{k}$. *Find* $\mathbf{u} \times \mathbf{v}$ *and verify (by computing the appropriate dot products) that* $\mathbf{u} \times \mathbf{v}$ *is perpendicular to both* $\mathbf{u}$ *and* $\mathbf{v}$.

$$\vec{u}\times\vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -3 \\ 1 & 0 & -6 \end{vmatrix} = \vec{i}\begin{vmatrix} 2 & -3 \\ 0 & -6 \end{vmatrix} - \vec{j}\begin{vmatrix} 0 & -3 \\ 1 & -6 \end{vmatrix} + \vec{k}\begin{vmatrix} 0 & 2 \\ 1 & 0 \end{vmatrix}$$

$$= -12\vec{i} - 3\vec{j} - 2\vec{k}$$

OR $\vec{u}\times\vec{v} = (2\vec{j} - 3\vec{k})\times(\vec{i} - 6\vec{k})$

$$= 2(\vec{j}\times\vec{i}) - 12(\vec{j}\times\vec{k}) - 3(\vec{k}\times\vec{i}) + 18\cancel{(\vec{k}\times\vec{k})}^{\,0}$$

$$= -2\vec{k} - 12\vec{i} - 3\vec{j} = -12\vec{i} - 3\vec{j} - 2\vec{k}$$

$$(\vec{u}\times\vec{v})\cdot\vec{u} = (-12)(0) + (-3)(2) + (-2)(-3) = -6 + 6 = 0$$

$$(\vec{u}\times\vec{v})\cdot\vec{v} = (-12)(1) + (-3)(0) + (-2)(-6) = -12 + 12 = 0$$