Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 50 points.

1. (8 pts.) Let \( \mathbf{r}(t) = (\sin(2t), \cos(2t), 2t), \ t \geq 0 \). Compute the unit tangent vector, \( \mathbf{T}(t) \), and the curvature, \( \kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} \), for the curve \( \mathbf{r}(t) \).

2. (8 pts.) Find \( f_x, f_y, f_{yy}, \) and \( f_{xy} \) for the function \( f(x, y) = x^3 \sin(4xy) \).

3. (4 pts. each) Let \( f(x, y) = 3x^2y - 2y^2x \).

   (a) Find \( \nabla f \).

   (b) Find the directional derivative of \( f \) at the point \( (1, 2) \) and in the direction \( \mathbf{v} = \mathbf{i} + 3\mathbf{j} \).

   (c) Find the maximum rate of change of \( f \) at the point \( (1, 2) \), and the direction in which \( f \) changes most rapidly at the point \( (1, 2) \). (Note: Direction should be in the form of a unit vector.)

   (d) Find the linearization of the function \( f(x, y) \) at the point \( (1, 2) \).

   (e) Use differentials to estimate the change in \( f \) when the point \( (1, 2) \) moves to the point \( (1.1, 2.3) \).

4. (8 pts.) Find all critical points of the function \( f(x, y) = x^3 + y^3 - 4x - 32y + 10 \) and use the Second Derivative Test to identify each as a local maximum, local minimum, or saddle point. (Hint: There are four critical points.)

5. (8 pts.) Use Lagrange multipliers to find the maximum and minimum values of \( f(x, y) = y^2 - 4x^2 \) subject to the constraint \( x^2 + 2y^2 = 4 \).