MATH 213 – 3 APRIL 2012 – EXAM 2

Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 50 points.

1. (8 pts.) Let $\mathbf{r}(t) = \langle \sin(2t), \cos(2t), 2t \rangle$, $t \geq 0$. Compute the unit tangent vector, $\mathbf{T}(t)$, and the curvature, $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|}$, for the curve $\mathbf{r}(t)$.

2. (8 pts.) Find f_x, f_y, f_{yy}, and f_{xy} for the function $f(x, y) = x^3 \sin(4xy)$.

3. (4 pts. each) Let $f(x, y) = 3x^2y - 2y^2x$.

(a) Find ∇f.

(b) Find the directional derivative of f at the point $(1, 2)$ and in the direction $\mathbf{v} = \mathbf{i} + 3\mathbf{j}$.

(c) Find the maximum rate of change of f at the point $(1, 2)$, and the direction in which f changes most rapidly at the point $(1, 2)$. (Note: Direction should be in the form of a unit vector.)

(d) Find the linearization of the function $f(x, y)$ at the point $(1, 2)$.

(e) Use differentials to estimate the change in f when the point $(1, 2)$ moves to the point $(1.1, 2.3)$.

4. (8 pts.) Find all critical points of the function $f(x, y) = x^3 + y^3 - 4x - 32y + 10$ and use the Second Derivative Test to identify each as a local maximum, local minimum, or saddle point. (Hint: There are four critical points.)

5. (8 pts.) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = y^2 - 4x^2$ subject to the constraint $x^2 + 2y^2 = 4$.