MATH 213 – 21 FEBRUARY 2012 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 75 points.

1. (5 pts. each) Let $\mathbf{a} = 3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$, and $\mathbf{b} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$.

   (a) Find the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (b) Find a unit vector parallel to $\mathbf{a}$.

   (c) Find $\mathbf{a} \times \mathbf{b}$.

   (d) Find the angle (in radians) between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b}$.

   (e) Find $\text{proj}_{\mathbf{b}}(\mathbf{a})$.

   (f) Write $\mathbf{a}$ as the sum of a vector parallel to $\mathbf{b}$ and a vector perpendicular to $\mathbf{b}$.

   (g) Find a vector function $\mathbf{r}(t)$ whose graph is the line with direction $\mathbf{a}$ and containing the point $(5, 0, -1)$.

2. (8 pts. each) Let $\mathbf{r}(t) = \langle 3 + t^3, 2 - 4t^3, 1 + 6t^3 \rangle$ be the position of an object at time t

   (a) Find the velocity, speed and acceleration of the object.

   (b) Prove that the trajectory of the object lies on a line in $\mathbf{R}^3$. (Hint: First find the unit tangent vector of $\mathbf{r}(t)$.)

3. (8 pts. each) Consider the vector–valued function $\mathbf{r}(t) = \cos(2t)\,\mathbf{i} + \sin(2t)\,\mathbf{j} + (4/3)t^{3/2}\,\mathbf{k}$.

   (a) Find $\mathbf{r}'(t)$, $\mathbf{r}''(t)$, and $|\mathbf{r}'(t)|$.

   (b) Find the arclength of the above curve for $1 \leq t \leq 2$.

4. (8 pts.) Suppose that the acceleration of a projectile is given by $\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{j} - 32\mathbf{k}$, and that its initial position is $\mathbf{r}(0) = \mathbf{0}$ and its initial velocity is $\mathbf{r}'(t) = 100(1/2\,\mathbf{i} + \sqrt{3}/2\,\mathbf{k})$. Find an expression for the position $\mathbf{r}(t)$ of the projectile at time t.