MATH 213 – 21 FEBRUARY 2012 – EXAM 1

Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 75 points.

1. (5 pts. each) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{b} = 3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}$.

   (a) Find the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (b) Write $\mathbf{a}$ as the product of its magnitude and its direction.

   (c) Find $\mathbf{a} \times \mathbf{b}$.

   (d) Find the sine of the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (e) Find $\text{proj}_{\mathbf{a}}(\mathbf{b})$.

   (f) Write $\mathbf{b}$ as the sum of a vector parallel to $\mathbf{a}$ and a vector perpendicular to $\mathbf{a}$.

   (g) Find a vector function $\mathbf{r}(t)$ whose graph is the line with direction $\mathbf{a}$ and containing the point $(6, 5, -1)$.

2. (8 pts. each) Let $\mathbf{r}(t) = \langle 3\sin t, 5\cos t, 4\sin t \rangle$ be the position of an object at time t

   (a) Find the velocity, speed and acceleration of the object.

   (b) Prove that the trajectory of the object lies on a sphere in $\mathbf{R}^3$.

3. (8 pts. each) Consider the vector–valued function $\mathbf{r}(t) = t\,\mathbf{i} + (1/3)t^{3/2}\,\mathbf{j} + t\,\mathbf{k}$.

   (a) Find $\mathbf{T}(t)$, the unit tangent vector of $\mathbf{r}(t)$.

   (b) Find the arclength of the above curve for $0 \leq t \leq 4$.

4. (8 pts.) Suppose that the acceleration of a projectile is given by $\mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{j} - 32\mathbf{k}$, and that its initial position is $\mathbf{r}(0) = \mathbf{0}$ and its initial velocity is $\mathbf{r}'(0) = 100(1/2\,\mathbf{i} + \sqrt{3}/2\,\mathbf{k})$. Find an expression for the position $\mathbf{r}(t)$ of the projectile at time t.