Answer each of the following questions. Show all work, as partial credit may be given. This exam will be counted out of a total of 75 points.

1. (5 pts. each) Let \( \mathbf{a} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}, \ \mathbf{b} = 3\mathbf{i} + 5\mathbf{j} - 4\mathbf{k}. \)

(a) Find the cosine of the angle between \( \mathbf{a} \) and \( \mathbf{b}. \)

(b) Write \( \mathbf{a} \) as the product of its magnitude and its direction.

(c) Find \( \mathbf{a} \times \mathbf{b}. \)

(d) Find the sine of the angle between \( \mathbf{a} \) and \( \mathbf{b}. \)

(e) Find \( \text{proj}_\mathbf{a}(\mathbf{b}). \)

(f) Write \( \mathbf{b} \) as the sum of a vector parallel to \( \mathbf{a} \) and a vector perpendicular to \( \mathbf{a}. \)

(g) Find a vector function \( \mathbf{r}(t) \) whose graph is the line with direction \( \mathbf{a} \) and containing the point \((6, 5, -1)\).

2. (8 pts. each) Let \( \mathbf{r}(t) = \langle 3 \sin t, 5 \cos t, 4 \sin t \rangle \) be the position of an object at time \( t \)

(a) Find the velocity, speed and acceleration of the object.

(b) Prove that the trajectory of the object lies on a sphere in \( \mathbb{R}^3. \)

3. (8 pts. each) Consider the vector–valued function \( \mathbf{r}(t) = t \mathbf{i} + (1/3)t^{3/2} \mathbf{j} + t \mathbf{k}. \)

(a) Find \( \mathbf{T}(t) \), the unit tangent vector of \( \mathbf{r}(t). \)

(b) Find the arclength of the above curve for \( 0 \leq t \leq 4. \)

4. (8 pts.) Suppose that the acceleration of a projectile is given by \( \mathbf{a}(t) = \mathbf{r}''(t) = \mathbf{j} - 32\mathbf{k}, \)
and that its initial position is \( \mathbf{r}(0) = \mathbf{0} \) and its initial velocity is \( \mathbf{r}'(0) = 100(1/2 \mathbf{i} + \sqrt{3}/2 \mathbf{k}). \)
Find an expression for the position \( \mathbf{r}(t) \) of the projectile at time \( t. \)