Review Session

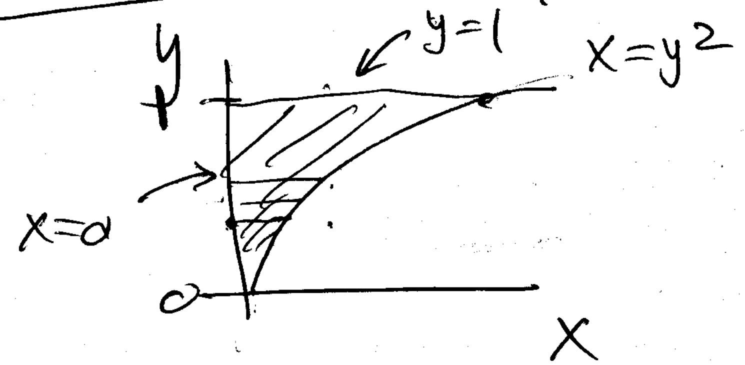
53, p 904 13.3
13.4 23, 15
14.4 13
Greens Theorem.
S06 #3, 4, 5
F09 #4

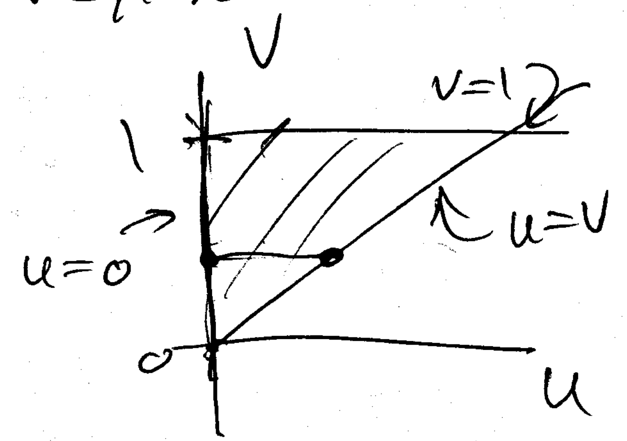
Spring 06 #3)

$$\int_0^1 \int_0^{y^2} 4xy \, dx\, dy$$

$x = u^2 \quad y = v$

① Change region.





$x = 0$: $u^2 = 0$, $u = 0$

$x = y^2$: $u^2 = v^2$, $u = v$ (since $u, v > 0$)

$y = 1$: $v = 1$

$$\int_{v=0}^{v=1} \int_{u=0}^{u=v} \underline{\qquad\qquad} \, du\, dv$$

② Change integrand : $4xy = 4u^2 v$

③ Change $dx\,dy$

Jacobian:
$x = u^2 \rightarrow \frac{\partial x}{\partial u} \quad \frac{\partial x}{\partial v} \rightarrow$
$y = v \rightarrow \frac{\partial y}{\partial u} \quad \frac{\partial y}{\partial v} \rightarrow$
$$\begin{vmatrix} 2u & 0 \\ 0 & 1 \end{vmatrix} = 2u$$

$$\int_0^1\int_0^{y^2} 4xy\,dx\,dy = \int_0^1\int_0^v 4u^2v \cdot 2u\,du\,dv.$$

$$= \int_0^1\int_0^v 8u^3v\,du\,dv.$$

#1 e) $f(x,y) = x^2y^3 + 2x^4y$

$$L(x,y) = f(1,2) + f_x(1,2)(x-1) + f_y(1,2)(y-2)$$

$$L(x,y,z) = F(x_0,y_0,z_0) + F_x(x_0,y_0,z_0)(x-x_0) + F_y(x_0,y_0,z_0)(y-y_0) + F_z(x_0,y_0,z_0)(z-z_0)$$

$$f(1,2) = (1)^2(2)^3 + 2(1)^4(2) = 12$$

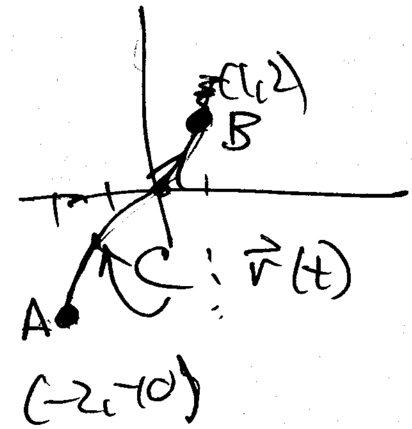
$f_x = 2xy^3 + 8x^3y \quad f_x(1,2) = 32$

$f_y = 3x^2y^2 + 2x^4 \quad f_y(1,2) = 14$

$$L(x,y) = 12 + 32(x-1) + 14(y-2)$$

#4) (a) $\int_C \nabla f \cdot d\vec{r}$ $\quad f(x,y) = xy + x^2$

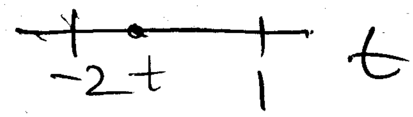
$C: \ y = x^3 + x$



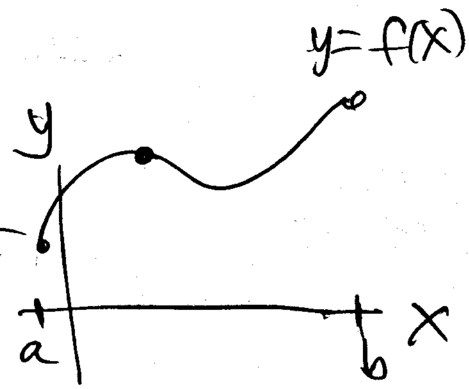
$$\int_C \nabla f \cdot d\vec{r} = f(B) - f(A)$$

$$= f(1,2) - f(-2,-10)$$

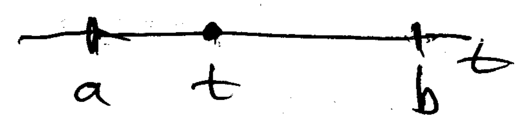
$$= 3 - 24 = -21$$



$\vec{r}(t) = \langle t, t^3 + t \rangle \quad -2 \le t \le 1$



$$\int_{-2}^{1} \underbrace{\langle t^3 + t + 2t, t \rangle}_{\nabla f(\vec{r}(t))} \cdot \underbrace{\langle 1, 3t^2 + 1 \rangle}_{\vec{r}\,'(t)} dt$$

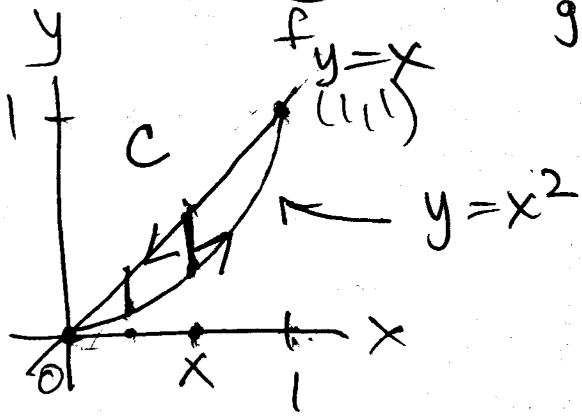


$\vec{r}(t) = \langle t, f(t) \rangle$

$\nabla f = \langle y + 2x, x \rangle$

$$= \int_{-2}^{1} (t^3 + 3t + 3t^3 + t)\, dt = \int_{-2}^{1} (4t^3 + 4t)\, dt$$

#5) $\vec{F} = \underbrace{xy}_{f}\,\vec{i} + \underbrace{\tfrac{1}{3}x^3}_{g}\,\vec{j}$



(a) $\displaystyle\int_C \vec{F}\cdot\vec{T}\,ds = \int_C \vec{F}\cdot d\vec{r} = \iint_R (g_x - f_y)\,dA$

$\displaystyle = \iint_R (x^2 - x)\,dA = \int_0^1\int_{x^2}^{x} (x^2 - x)\,dy\,dx$

(b) $\displaystyle\int_C \vec{F}\cdot\vec{n}\,ds = \iint_R (f_x + g_y)\,dA$

$\displaystyle = \int_0^1\int_{x^2}^{x} (y + 0)\,dy\,dx = \int_0^1\int_{x^2}^{x} y\,dy\,dx$

#4b) $\vec{F} = (x^2+y^2)\vec{i} + (2xy+3y^2)\vec{j}$

Find f so that $\vec{F} = \nabla f$

$f_x = x^2+y^2 \longrightarrow f = \frac{1}{3}x^3 + xy^2 + g(y)$

$f_y = 2xy+3y^2 \qquad f_y = 2xy + g'(y)$

$\therefore g'(y) = 3y^2$

$g(y) = y^3 + c \qquad f = \frac{1}{3}x^3 + xy^2 + y^3 + c.$

$f = \frac{1}{3}x^3 + xy^2 + \_\_\_\_ \qquad f = xy^2 + y^3 + \_\_\_\_$

$f = \frac{1}{3}x^3 + xy^2 + y^3$

~~Fall 09 #4 $f(x,y) = x^2 + 2y^2 + x^2y$~~

Spring 06 #2 $f(x,y) = 2x^3 + y^3 - 3x^2 - 12x - 3y$

crit pts: $f_x = 6x^2 - 6x - 12 \quad f_y = 3y^2 - 3$

$6x^2 - 6x = 12 \longrightarrow x^2 - x = 2 \rightarrow x^2 - x - 2 = 0$

$3y^2 = 3 \longrightarrow y = \pm 1 \qquad (x+1)(x-2) = 0$

CP: $(-1,1), (-1,-1), (2,1), (2,-1) \qquad x=-1 \quad x=2$

$D(x,y) = f_{xx}f_{yy} - (f_{xy})^2 = (12x-6)(6y)$

$f_{xx} = 12x-6 \quad f_{yy} = 6y \quad f_{xy} = 0$

① $(-1,1)$: $D(-1,1) < 0$ saddle

② $(-1,-1)$: $D(-1,-1) > 0$ local max
$f_{xx}(-1,-1) = -18 < 0$

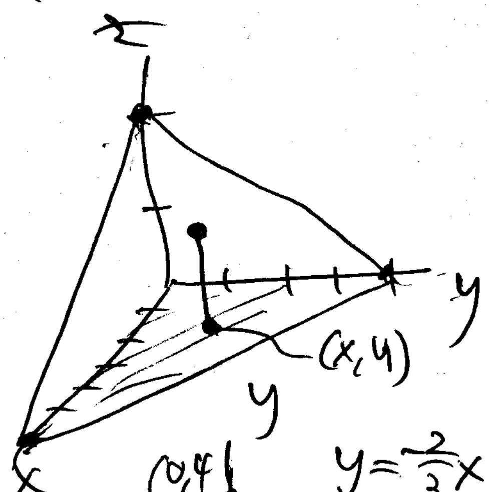
③ $(2,1)$: $D(2,1) > 0$ local min.
$f_{xx}(2,1) > 0$

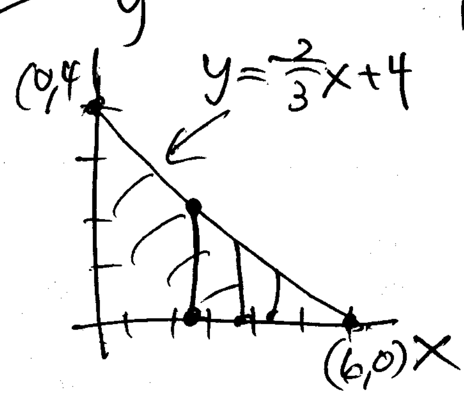
④ $(2,-1)$: $D(2,-1) < 0$ saddle.

13.4 #15 $2x+3y+6z=12 \rightarrow 6z = 12-2x-3y$

$z = 2 - \frac{1}{3}x - \frac{1}{2}y$

$$\iiint_D 1\, dV$$

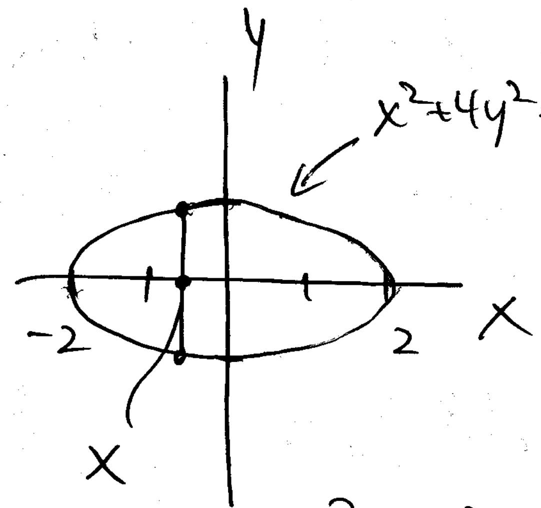




$$\int_0^6 \int_0^{-\frac{2}{3}x+4} \int_0^{2-\frac{1}{3}x-\frac{1}{2}y} 1\, dz\, dy\, dx$$

23) $x^2+4y^2=4$

$z=3-x \qquad z=x-3$



$$\int_{-2}^{2}\int_{-(1-\frac{x^2}{4})^{1/2}}^{(1-\frac{x^2}{4})^{1/2}}\int_{x-3}^{3-x} 1\,dz\,dy\,dx$$

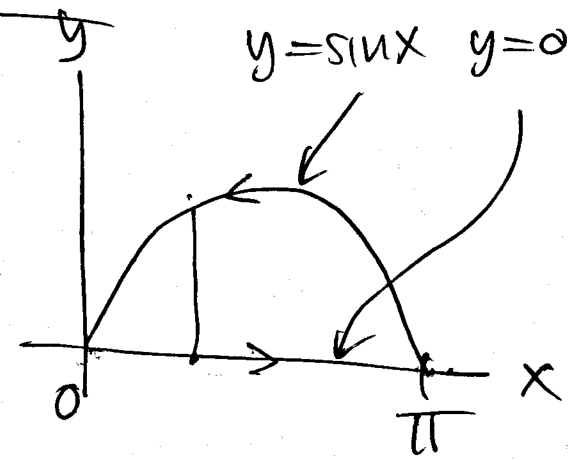
$x^2+4y^2=4$

$4y^2=4-x^2$

$y^2=1-\frac{x^2}{4}$

$y=\pm\sqrt{1-\frac{x^2}{4}}$

14.4 #13) $\vec{F}=\langle 2y,-2x\rangle$



$$\int_C \vec{F}\cdot d\vec{r}=\iint_R (g_x-f_y)\,dA$$

$$=\int_0^{\pi}\int_0^{\sin x}(-2-2)\,dy\,dx=\int_0^{\pi}\int_0^{\sin x}(-4)\,dy\,dx$$