11Z 13 — pick up at George's office

Monday 5-7 Review Session (George)
12-1^{15} in ST 1, 242

Tuesday 5-8 Review Session (Walnut)
10^{30}-11^{45} in this room.

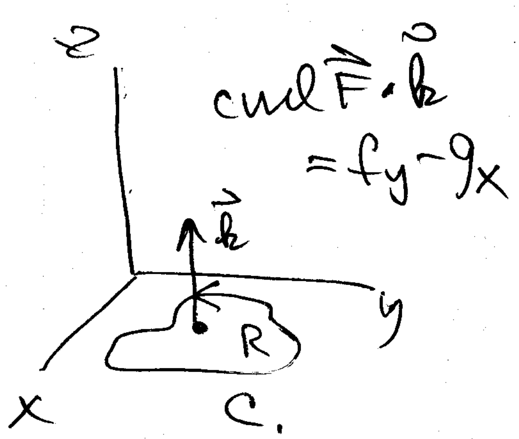
Final Thursday 5-10, 10^{30}-1^{15} pm.

Stokes' Theorem

$$\oint_C \vec{F} \cdot d\vec{r} = \iint_S (\text{curl}\, \vec{F} \cdot \vec{n})\, dS$$

circulation around C

surface integral of curl on S



Green's Thm

Stokes' generalizes Green's Thm.

Divergence Thm

$$\iint_S (\vec{F} \cdot \vec{n})\, dS = \iiint_D (\operatorname{div} \vec{F})\, dV.$$

flow across surface S

triple integral of divergence over region bounded by S.

<u>14.6</u> Surface Integrals:

A. Parametrized surface.

$\vec{r}(t) = \langle x(t), y(t) \rangle$

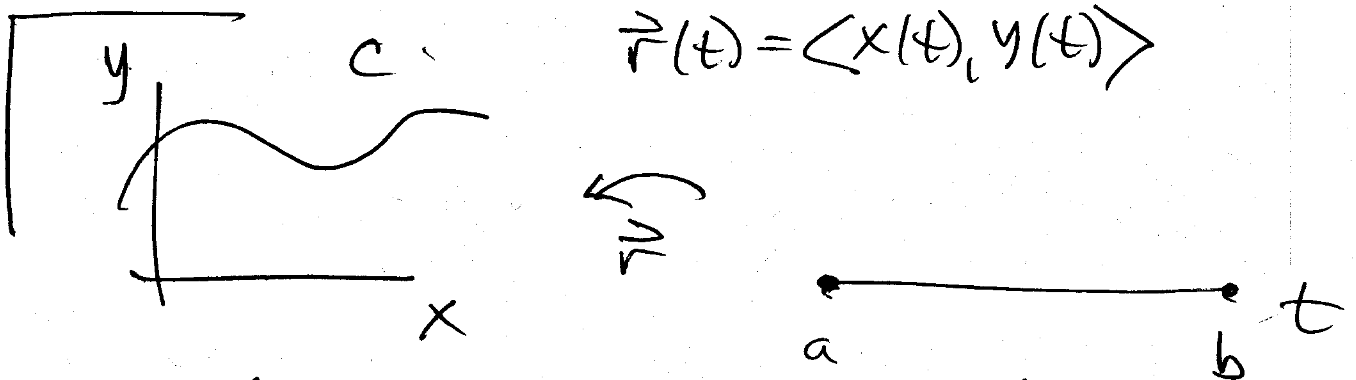
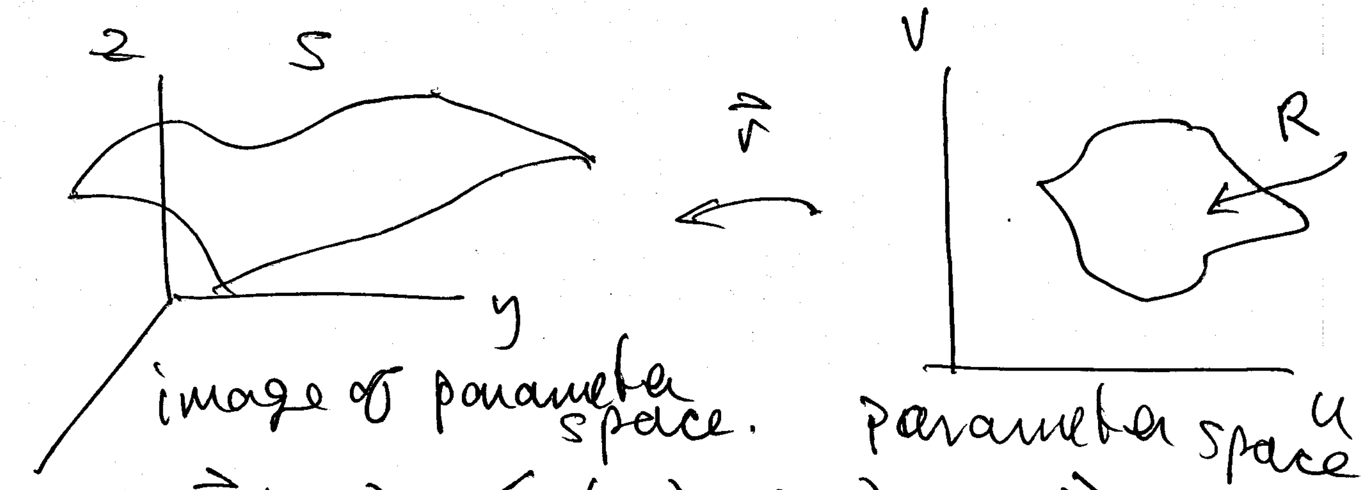


image of parameter space.

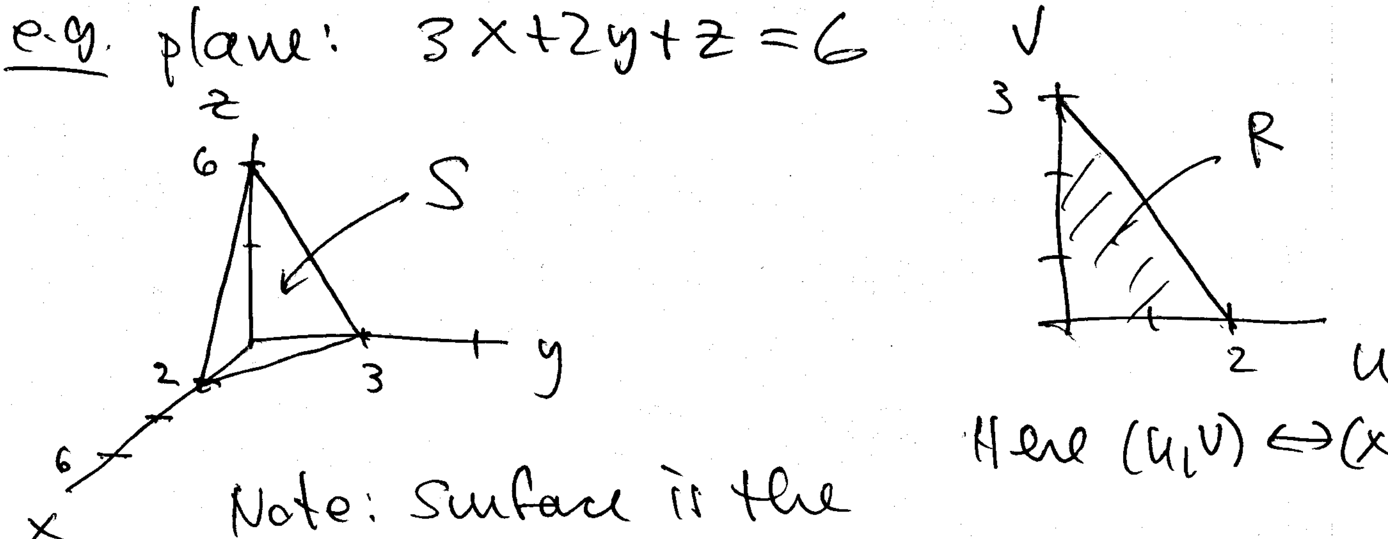
parameter space

What would a surface in $\mathbb{R}^3$ look like?



$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

e.g. plane: $3x+2y+z=6$

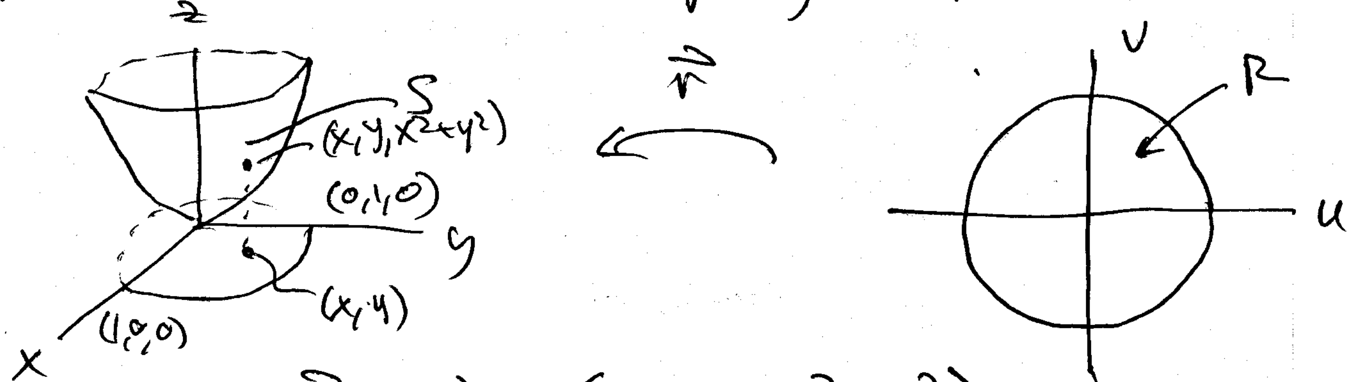


Here $(u,v) \Leftrightarrow (x,y)$.

Note: Surface is the graph of $z=6-3x-2y$.

$$\vec{r}(u,v) = \langle u, v, 6-3u-2v \rangle$$

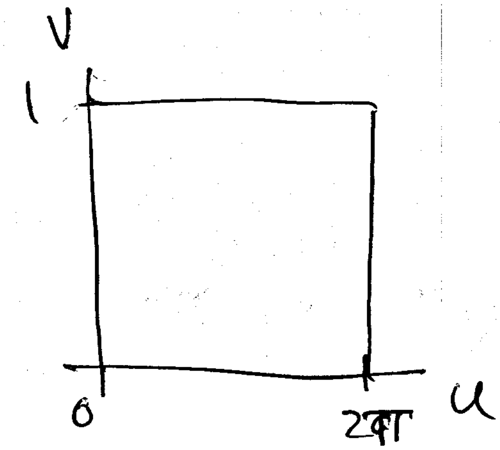
paraboloid: $z=x^2+y^2$, $x^2+y^2 \le 1$



$$\vec{r}(u,v) = \langle u, v, u^2+v^2 \rangle$$

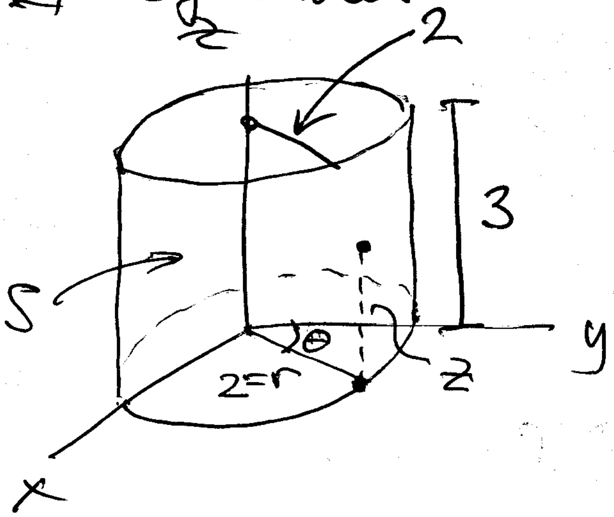
OR use polar coords.

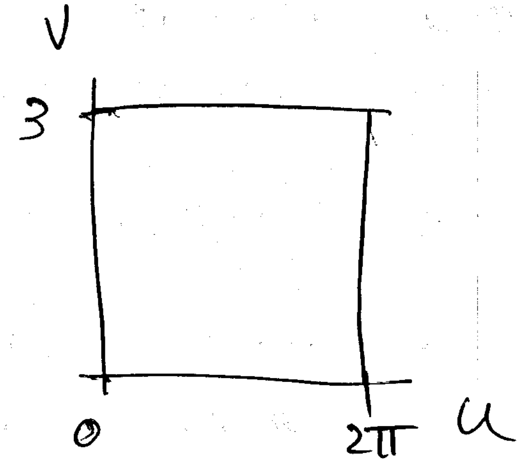
$$\vec{r}(u,v) = \langle v\cos u, v\sin u, v\rangle$$



$(u,v) \leftrightarrow (\theta, r)$

eg cylinder

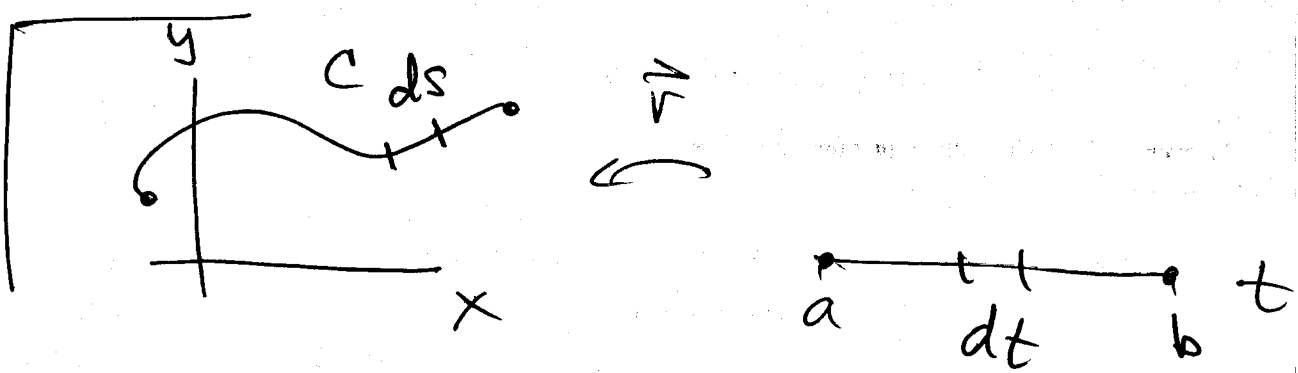




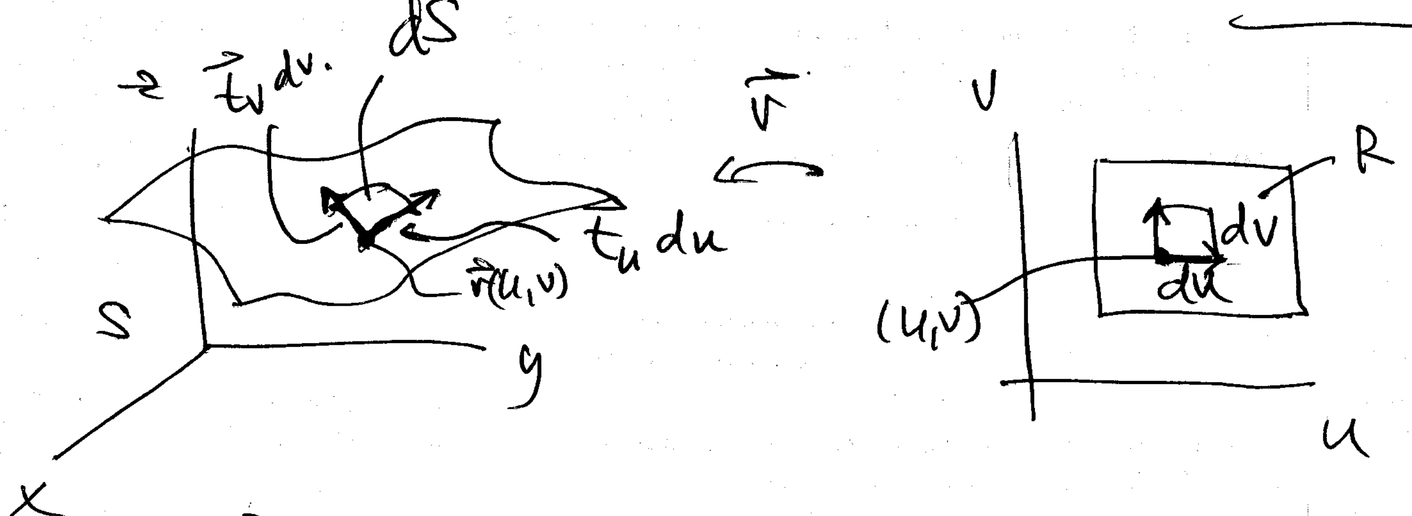
$(u,v) \leftrightarrow (\theta, z)$

$$\vec{r}(u,v) = \langle 2\cos u, 2\sin u, v\rangle$$

B. Surface area element dS



$$ds = |\vec{r}\,'(t)|\,dt \quad (\text{Note: } \vec{r}\,'(t) \text{ tangent to } C)$$



$$\vec{r}(u,v) = \langle x(u,v), y(u,v), z(u,v) \rangle$$

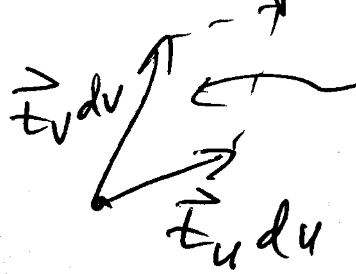
Idea: $\dfrac{\partial \vec{r}}{\partial u} = \left\langle \dfrac{\partial x}{\partial u}, \dfrac{\partial y}{\partial u}, \dfrac{\partial z}{\partial u} \right\rangle = \vec{t}_u$

$$dx = \frac{\partial x}{\partial u}\,du \quad dy = \frac{\partial y}{\partial u}\,du \quad dz = \frac{\partial z}{\partial u}\,du$$

vector $\left\langle \dfrac{\partial x}{\partial u}, \dfrac{\partial y}{\partial u}, \dfrac{\partial z}{\partial u} \right\rangle du = \vec{t}_u\,du$

Similarly: $\vec{t}_v\,dv = \left\langle \dfrac{\partial x}{\partial v}, \dfrac{\partial y}{\partial v}, \dfrac{\partial z}{\partial v} \right\rangle dv.$

I have a parallelogram in $\mathbb{R}^3$.



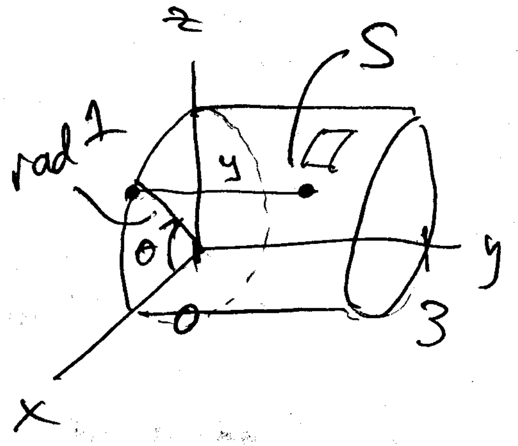
$$dS = |\vec{t}_u du \times \vec{t}_v dv| = |\vec{t}_u \times \vec{t}_v| \, du \, dv.$$

Also note: $\vec{t}_u \times \vec{t}_v$ is normal to surface S.

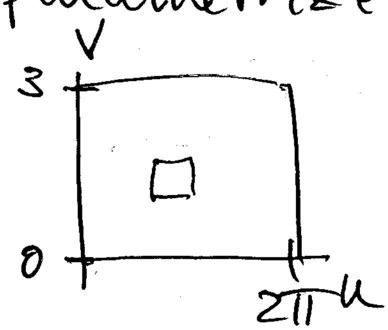
So $\vec{n} = \dfrac{\vec{t}_u \times \vec{t}_v}{|\vec{t}_u \times \vec{t}_v|}$ = unit normal vector to S.

eg 29) $\displaystyle\iint_S f(x,y,z)\, dS$

$f(x,y,z) = x \quad S: x^2 + z^2 = 1, \; 0 \le y \le 3.$



① parametrize S.



$(u,v) \Leftrightarrow (\theta, y)$

$\vec{r}(u,v) = \langle \cos u, v, \sin u \rangle$

② $f(x,y,z) = f(\cos u, v, \sin u) = \cos u$

③ $dS = \underbrace{|\vec{t}_u \times \vec{t}_v|}\, du\, dv$

$$\vec{t}_u = \frac{\partial \vec{r}}{\partial u} = \left\langle \frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u} \right\rangle = \langle -\sin u, 0, \cos u \rangle$$

$$\vec{t}_v = \frac{\partial \vec{r}}{\partial v} = \left\langle \frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v} \right\rangle = \langle 0, 1, 0 \rangle$$

$$\vec{t}_u \times \vec{t}_v = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sin u & 0 & \cos u \\ 0 & 1 & 0 \end{vmatrix} = -\cos u\, \vec{i} + -\sin u\, \vec{k}$$

$$= \langle -\cos u, 0, -\sin u \rangle$$

$$|\vec{t}_u \times \vec{t}_v| = |\langle -\cos u, 0, -\sin u \rangle| = 1$$

$$\iint_S f(x,y,z)\, dS = \int_0^{2\pi} \int_0^3 \cos u\,(1)\, dv\, du$$

etc....