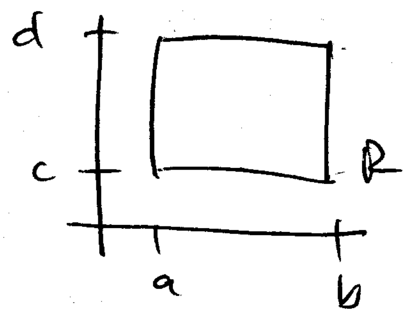
Exam 2 - Tues 4/3 11.9, 12.1-12.9

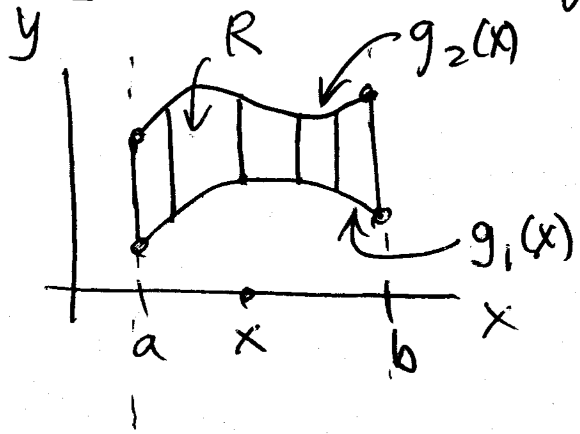
Quiz 9 - Tues 4/3 13.1.

Double Integrals



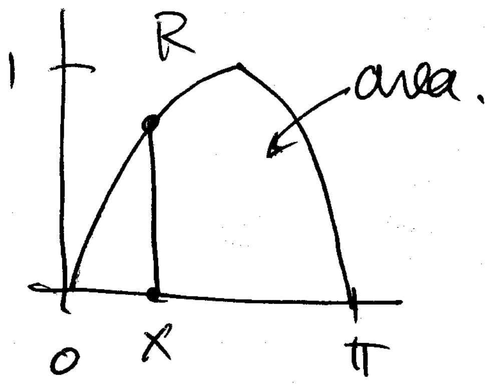
$$\iint_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy$$

13.2 General Regions



$$\iint_R f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx$$

e.g.

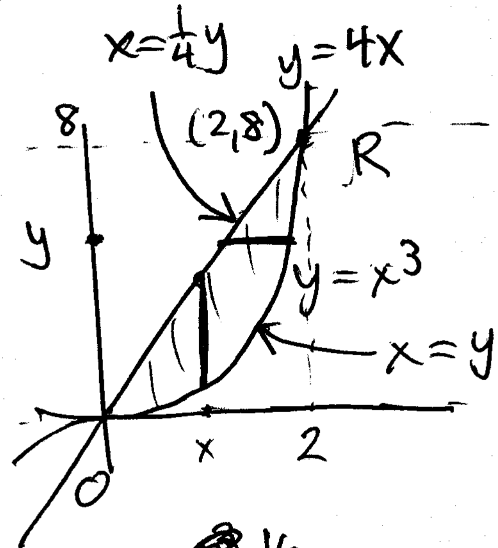


$y = \sin x$

$$\iint_R dA = \int_0^{\pi}\left[\int_0^{\sin x} 1\,dy\right]dx$$

$$= \int_0^{\pi}\left(y\Big|_0^{\sin x}\right)dx = \int_0^{\pi} \sin x\,dx$$

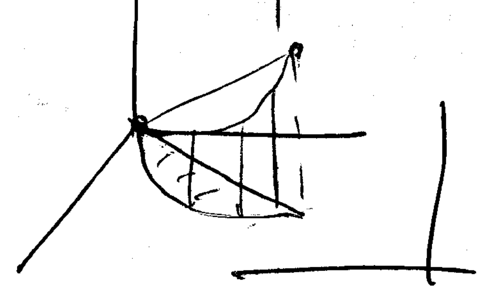
eg #7)



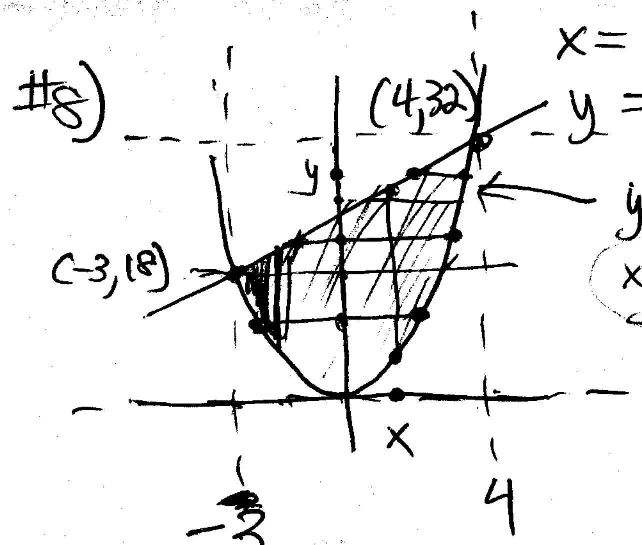
$$\iint_R f(x,y)\,dA = \int_0^{2}\int_{x^3}^{4x} f(x,y)\,dy\,dx$$

$$= \int_0^{8}\int_{\frac{1}{4}y}^{y^{1/3}} f(x,y)\,dx\,dy$$

$$\int_0^{2}\int_{x^3}^{4x}(x^2+y^2)\,dy\,dx$$



#8)



$$\iint_R f(x,y)\, dA$$

$$\int_{-3}^{4}\int_{2x^2}^{2x+24} f(x,y)\, dy\, dx$$

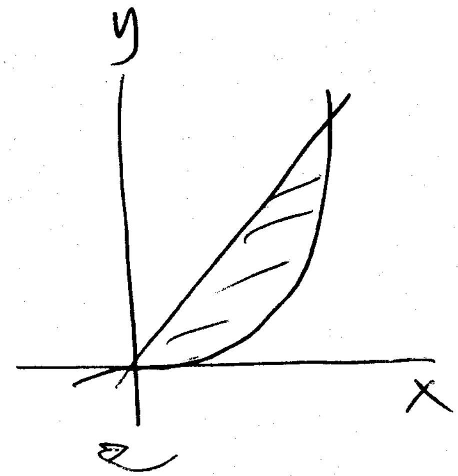
$2x^2 = 2x + 24$

$x^2 - x - 12 = 0$

$(x+3)(x-4) = 0$

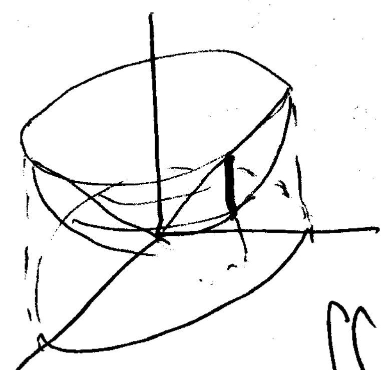
$x = -3$. $x = 4$

$$\cancel{\int_0^{32}\left[\int \quad \right] dy}$$



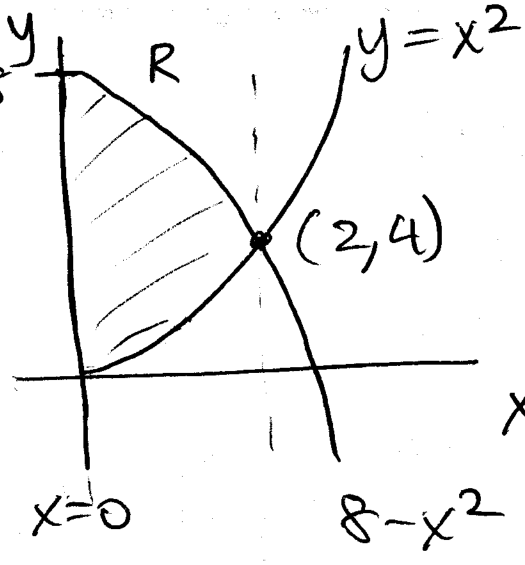
$$= \int_{18}^{32}\int_{\frac{y-24}{2}}^{\left(\frac{y}{2}\right)^{1/2}} f(x,y).\, dx\, dy$$

$$+ \int_{0}^{18}\int_{-\left(\frac{y}{2}\right)^{1/2}}^{\left(\frac{y}{2}\right)^{1/2}} f(x,y).\, dx\, dy$$



$$\iint_D \frac{?}{\quad}\, dA$$

#20) $\iint\limits_R (x+y)\,dA$



$$= \int_0^2 \left[\int_{x^2}^{8-x^2} (x+y)\ dy\right] dx$$

$$= \int_0^2 \left(xy + \frac{1}{2}y^2 \Big|_{x^2}^{8-x^2}\right) dx$$

$x^2 = 8-x^2$

$2x^2 = 8$

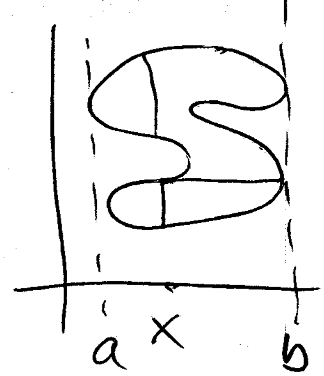
$x^2 = 4$

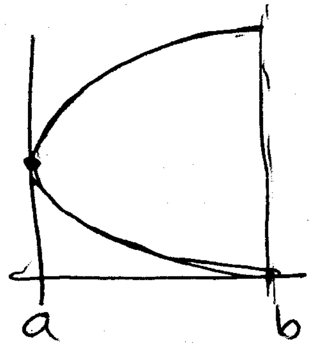
$x = \pm 2$

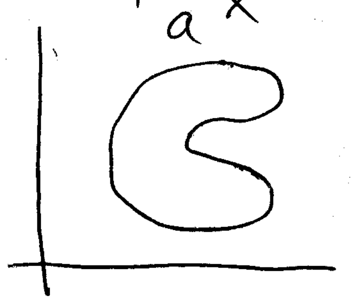
$$= \int_0^2 x(8-x^2) + \frac{1}{2}\underbrace{(8-x^2)^2}_{64-16x^2+x^4} - x \cdot x^2 - \frac{1}{2}(x^2)^2\ dx$$

$$= \int_0^2 8x - x^3 + 32 - 8x^2 + \cancel{\frac{1}{2}x^4} - x^3 - \cancel{\frac{1}{2}x^4}\ dx$$

$$= \int_0^2 (-2x^3 - 8x^2 + 8x + 32)\,dx$$





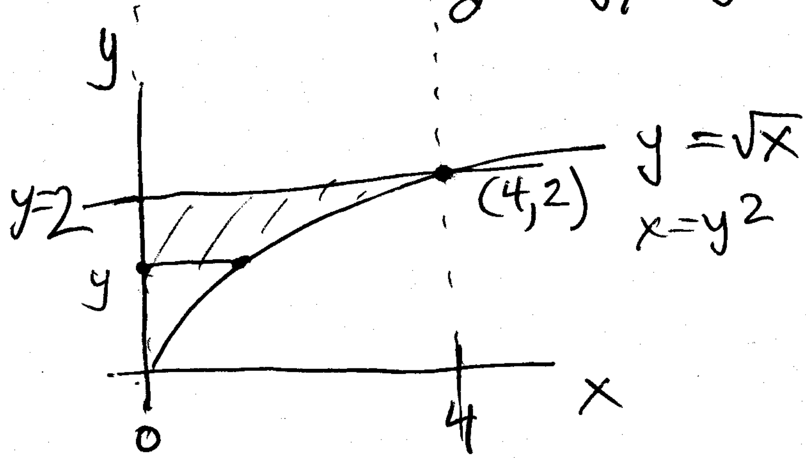


$$= -\frac{1}{2}x^4 - \frac{8}{3}x^3 + 4x^2 + 32x \Big|_0^2$$

$$= -8 - \frac{64}{3} + 16 + 64 - 0 = 72 - \frac{64}{3} = \frac{152}{3}$$

~~216~~ ~~64~~ ~~152~~

#52) $\displaystyle\int_0^4 \int_{\sqrt{x}}^2 \frac{x}{y^5+1}\,dy\,dx$



$$= \int_0^2 \left[\int_0^{y^2} \frac{x}{y^5+1}\,dx \right] dy$$

$$= \int_0^2 \left(\frac{1}{2} \cdot \frac{x^2}{y^5+1} \Big|_0^{y^2} \right) dy = \int_0^2 \frac{1}{2}\left(\frac{y^4}{y^5+1} - 0 \right) dy$$

$$= \frac{1}{2} \cdot \frac{1}{5} \int_0^2 \frac{5y^4}{y^5+1}\,dy = \frac{1}{10}\int_1^{33} \frac{du}{u} = \frac{1}{10} \ln|u| \Big|_1^{33}$$

$$= \frac{1}{10} \ln(33)$$

$u = y^5 + 1$

$du = 5y^4\,dy$

$y = 0 \rightarrow u = 1$

$y = 2 \rightarrow u = 33$