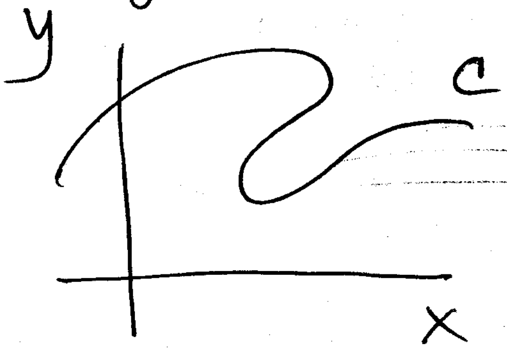
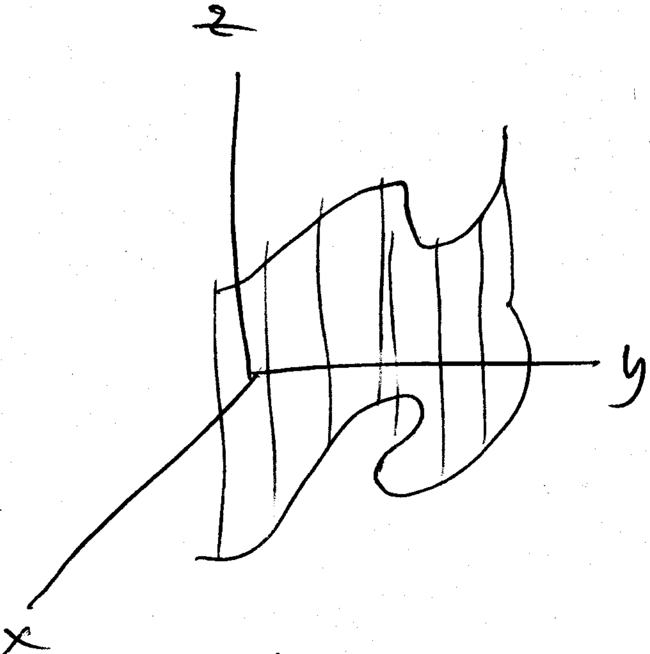
Plane: $ax+by+cz=d$ (linear equation)
In general, a surface has an equation of the form $F(x,y,z)=0$.

More general surfaces.

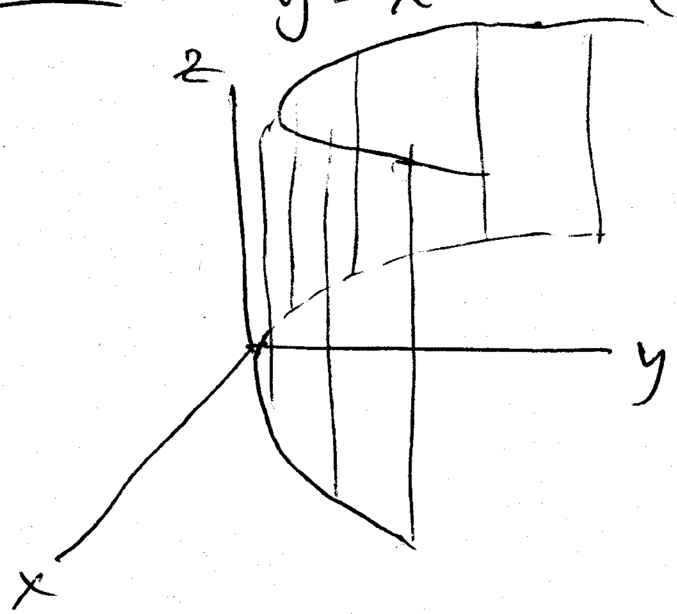
B. Cylinders.



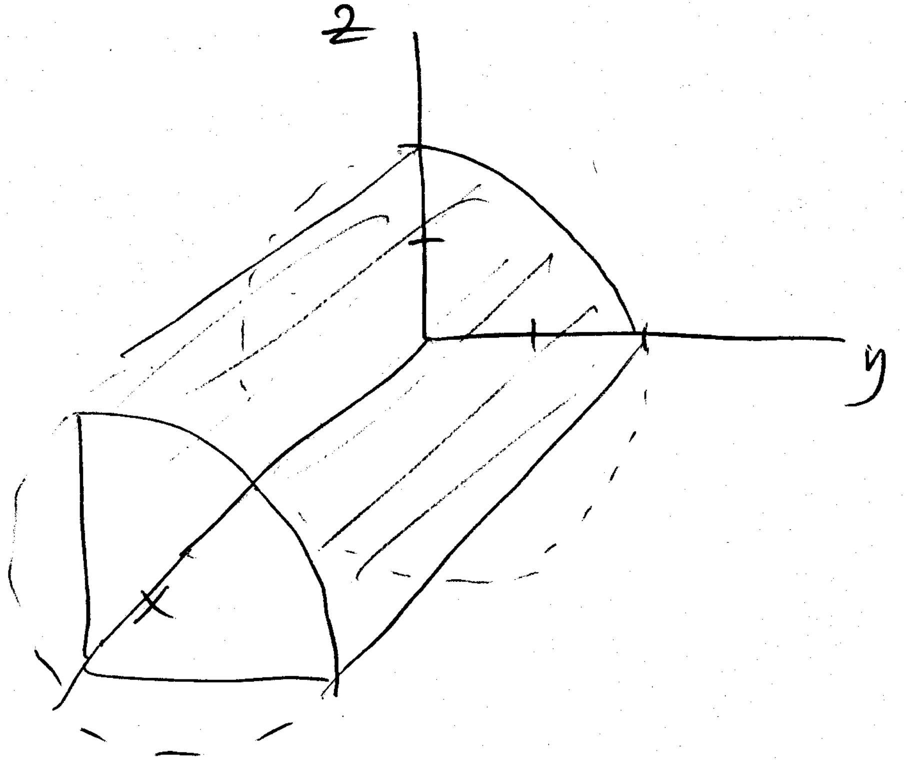


If curve is given by $f(x,y)=0$, then the surface is also given by $f(x,y)=0$.

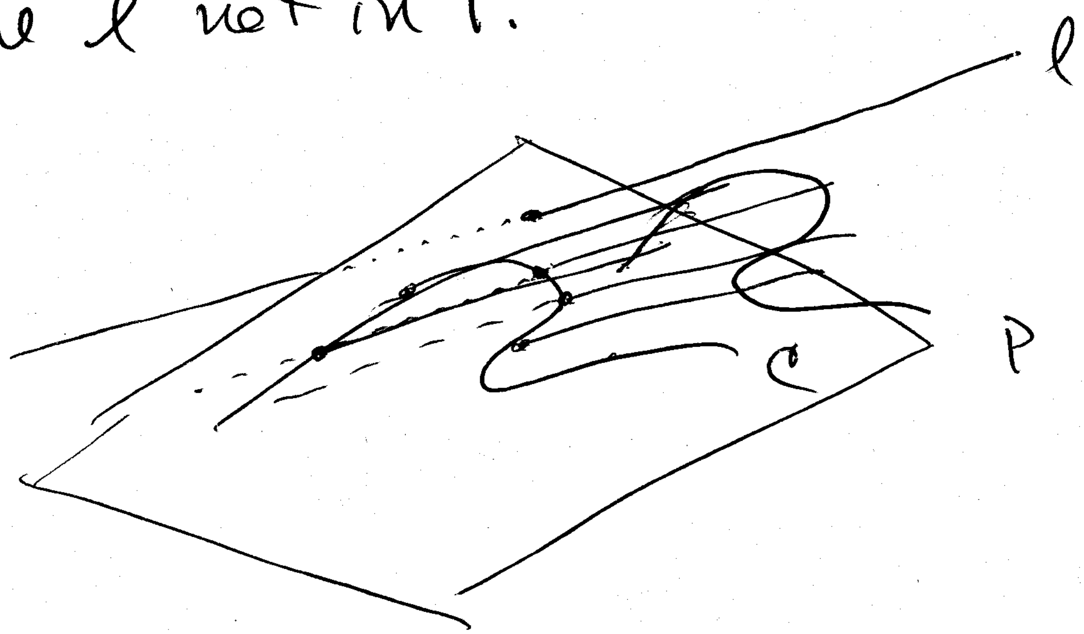
e.g. $y=x^2$ $\left(\underbrace{y-x^2}_{f(x,y)}=0\right)$



eg $y^2+z^2=4$



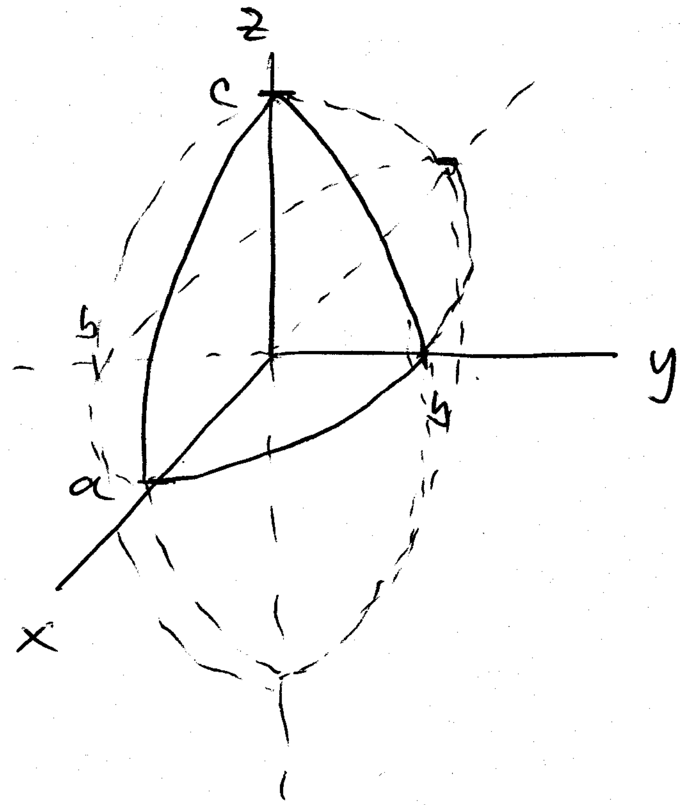
In general: a <u>cylinder</u> is given by a plane P and a curve C in P and a line ℓ not in P.



C. Quadric surfaces.

Given by a quadratic equation in x, y, z.

a. $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ ellipsoid



sketch traces

x-y plane: $z=0$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipse

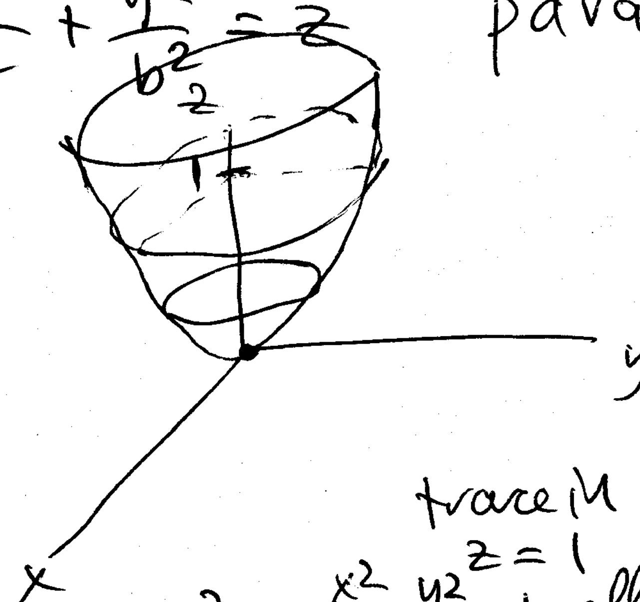
y-z plane: $x=0$

$\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

x-z plane: $y=0$

$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$

b. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=z$ paraboloid



x-y trace. $z=0$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$

y-z trace

$\frac{y^2}{b^2}=z$

x-z trace

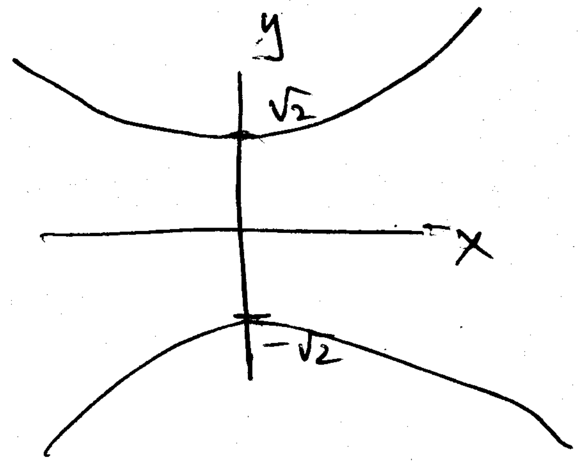
$\frac{x^2}{a^2}=z$

trace in $z=1$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ellipse pg #46) $$\frac{y^2}{2} + \frac{z^2}{36} - 4x^2 = 1$$

x - y trace

$\frac{y^2}{2} - 4x^2 = 1$ hyperbola

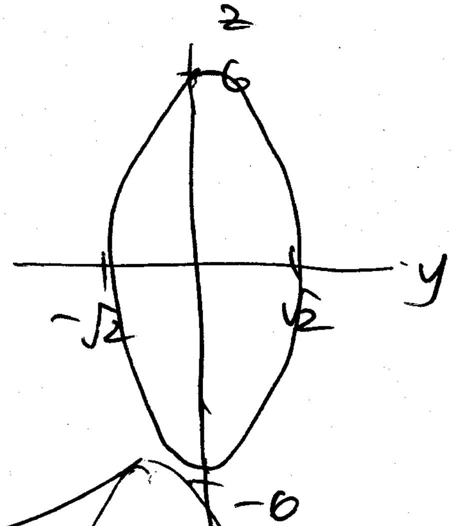


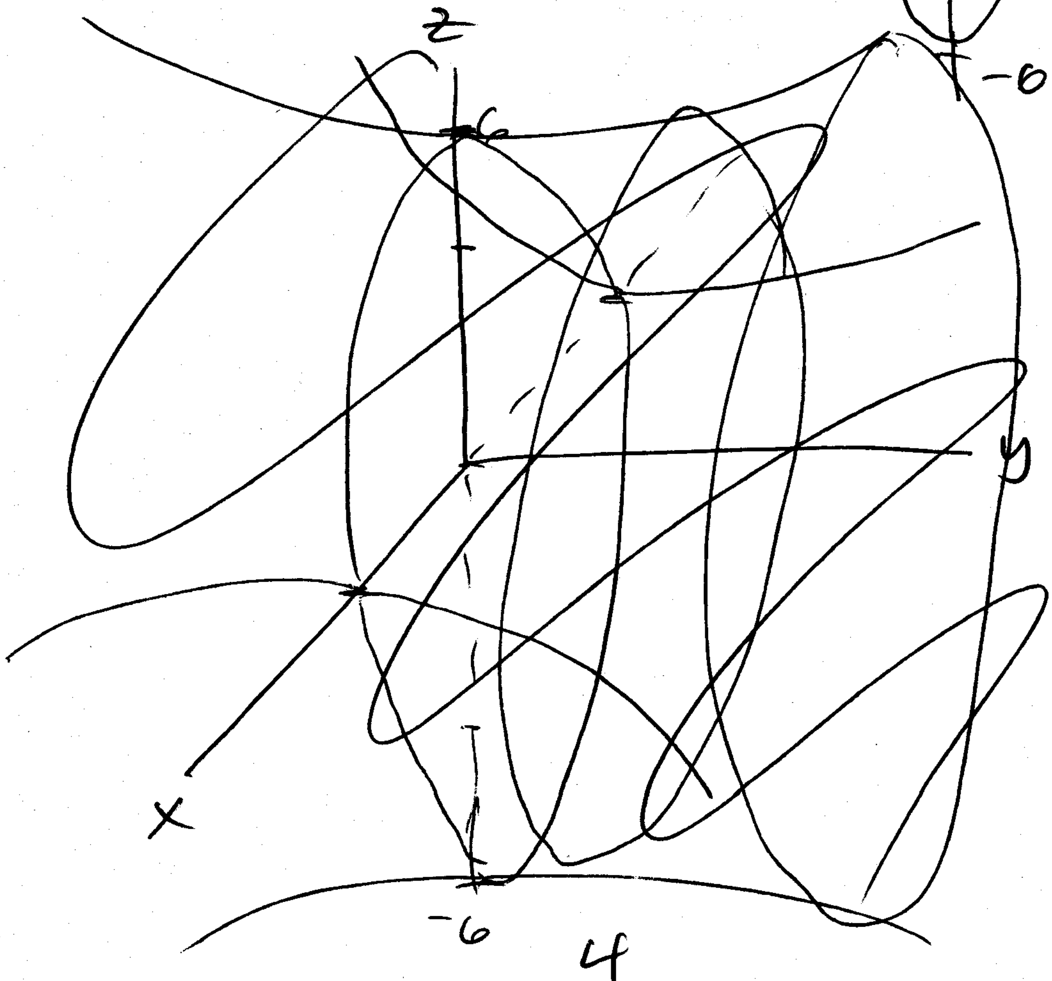
x - z trace

$\frac{z^2}{36} - 4x^2 = 1$ hyperbola.

y - z plane

$\frac{y^2}{2} + \frac{z^2}{36} = 1$ ellipse





oops!

eg #58) $y^2 + \frac{z^2}{2} - 4x^2 = -1$

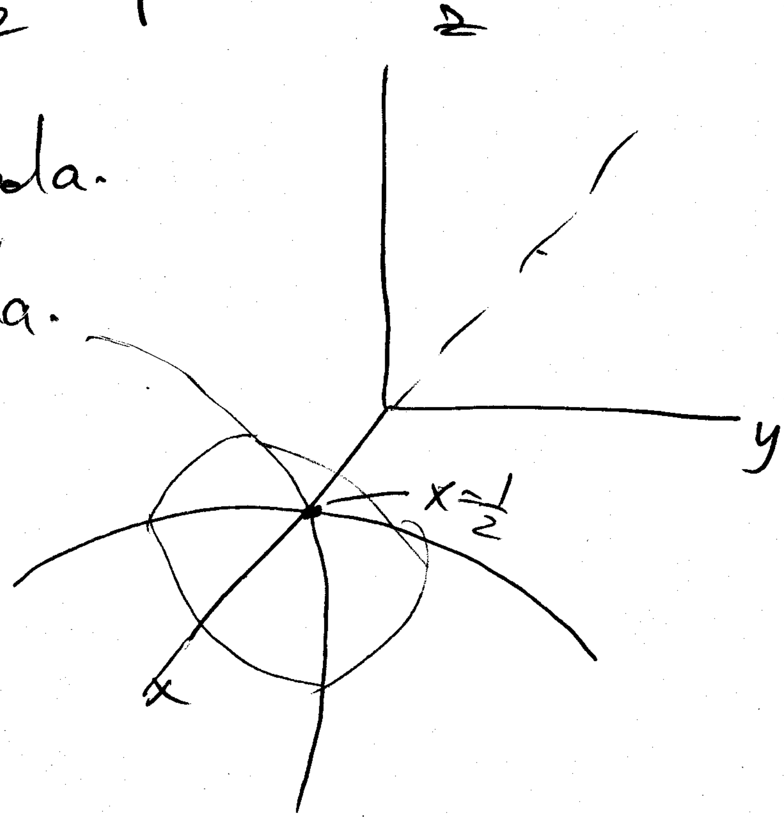
$$4x^2 - y^2 - \frac{z^2}{2} = 1$$

traces:

xy plane. hyperbola.

xz plane. hyperbola.

yz plane.



Look at traces in planes $x=c$.

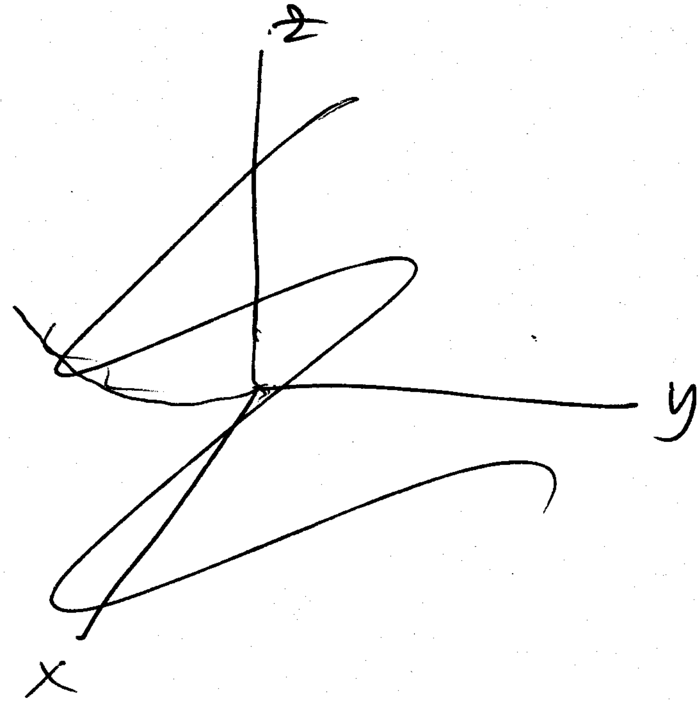
$$-y^2 - \frac{z^2}{2} = 1 - 4c^2$$

$$y^2 + \frac{z^2}{2} = 4c^2 - 1$$

If $4c^2 - 1 > 0$ $(c > \frac{1}{2})$ then ellipses.

e.g. #50 hyperbolic paraboloid (saddle).

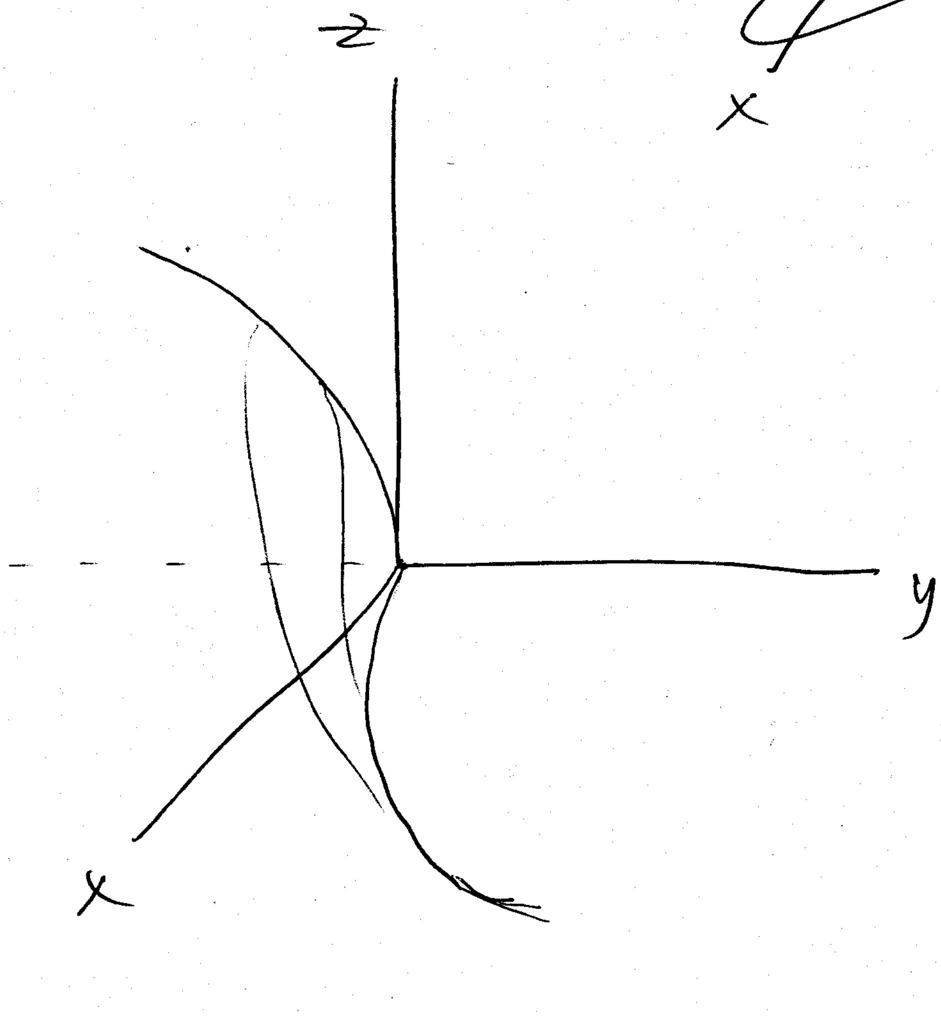
$$y = \frac{x^2}{16} - 4z^2$$



x-y trace. parabola.

$$y = \frac{x^2}{16}$$

y-z trace.



12-2 Graphs and Level curves.

We have seen: $\vec{r}(t) : \mathbb{R} \to \mathbb{R}^3$

Now looking at: real #s, vectors.

$f : \mathbb{R}^3 \longrightarrow \mathbb{R}$

point in space, real #s

Say f is a function of 3 variables (or 2 variables)

Write $z = f(x,y)$ or $w = f(x,y,z)$.

A. Domain.

e.g. $f(x,y) = (x-4y^2)^{1/2}$

Domain:

$f(1,0) = 1$

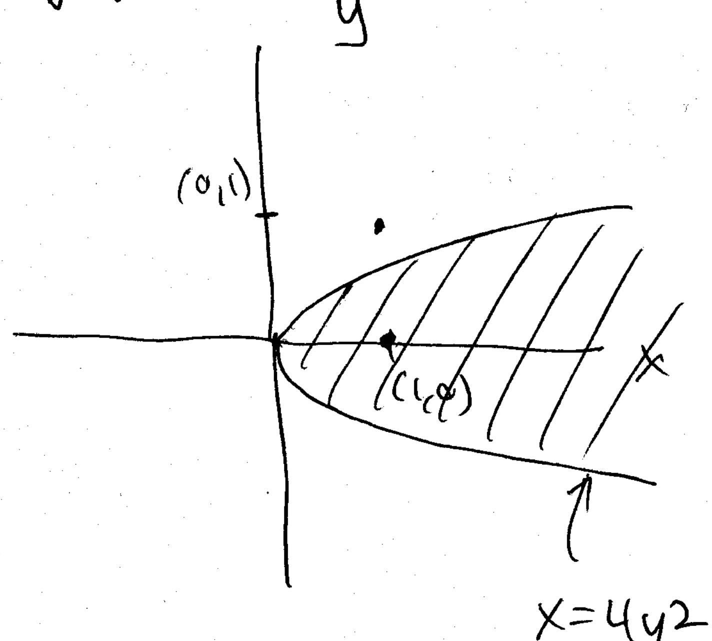
$f(0,1) = (-4)^{1/2}$ not defined

$f(1,1) = (1-4)^{1/2} = (-3)^{1/2}$

Need: $x - 4y^2 \geq 0$

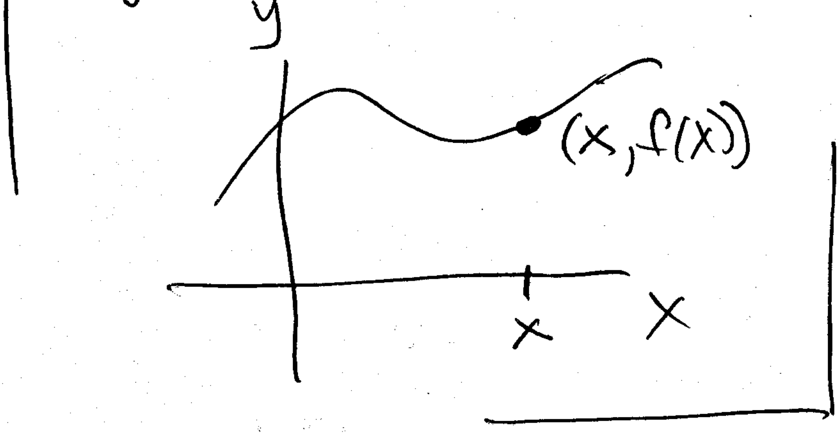
$x \geq 4y^2$

$x = 4y^2$

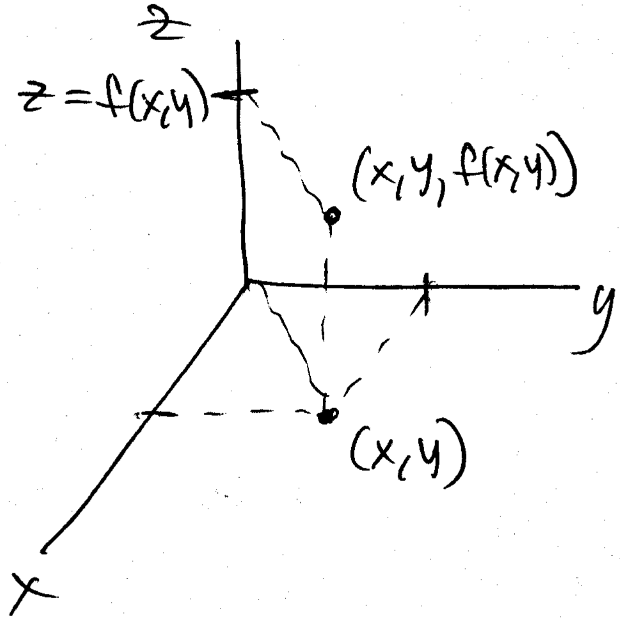


B. Graphs.

$y = f(x)$



$z = f(x,y)$



For graph of $z = f(x,y)$ to be a plane, would need: $f(x,y) = ax + by + d$

graph is a surface in $\mathbb{R}^3$.

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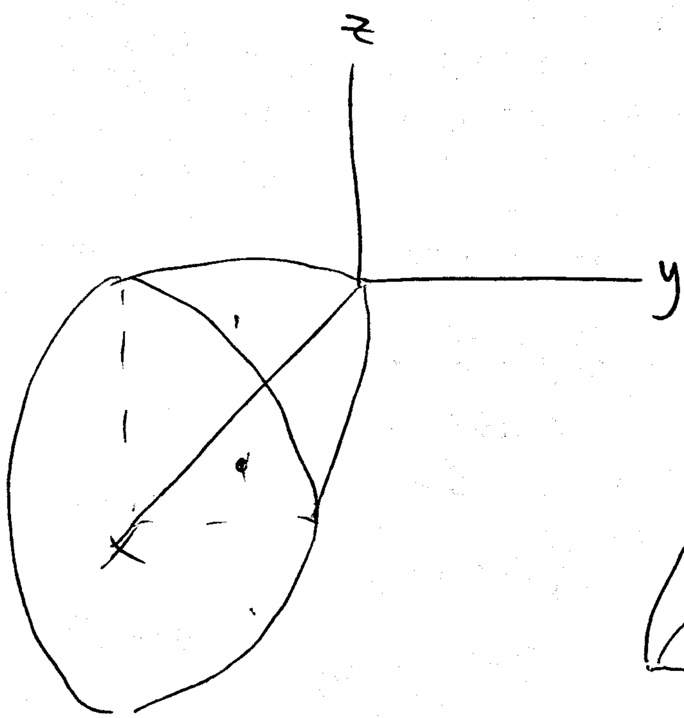
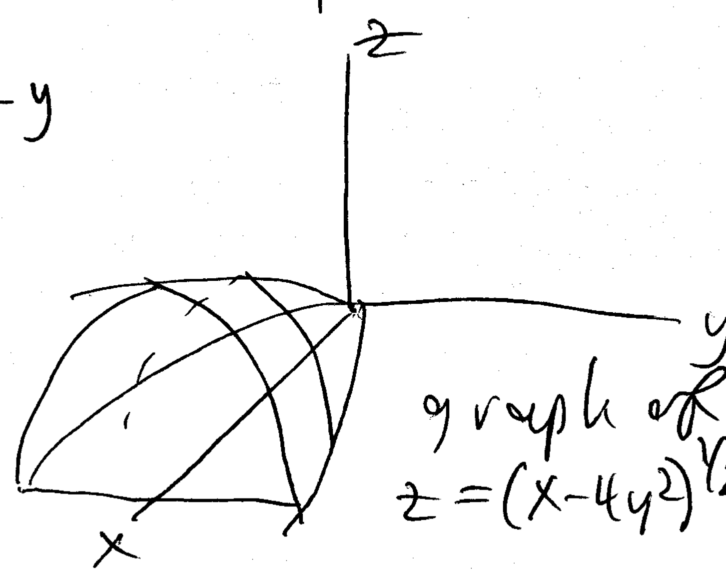
eg $z = (x - 4y^2)^{1/2} \longrightarrow z^2 = x - 4y^2$

$z^2 + 4y^2 = x$

paraboloid





graph of $z = (x - 4y^2)^{1/2}$.

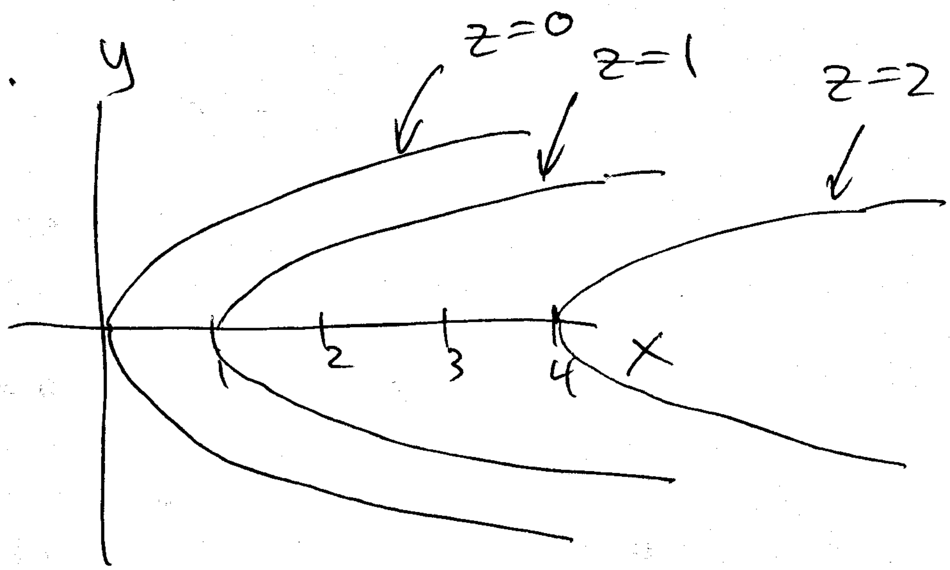
c. Level curves. (or level surfaces).

$z = f(x,y)$

A level curve of $f(x,y)$ is a curve of the form $f(x,y) = c$ some const. c.

e.g. $z = (x-4y^2)^{1/2}$

level curves.



$z=0$:

$0 = (x-4y^2)^{1/2}$

$0 = x-4y^2$

$x = 4y^2$

$z=1$:

$1 = (x-4y^2)^{1/2}$

$x-4y^2 = 1$

$x = 4y^2+1$

$z=2$:

$2 = (x-4y^2)^{1/2}$

$x = 4y^2+4$